03.1 ss.svar State variable system representation

1 **State variables**, typically denoted x_i, are members of a minimal set of variables that completely expresses the **state** (or status) of a system. All variables in the system can be expressed algebraically in terms of state variables and **input variables**, typically denoted u_i.

2 A **state-determined system model** is a system for which

1. a mathematical description in terms of n state variables x_i,
2. initial conditions $x_i(t_0)$, and
3. inputs $u_i(t)$ for $t \geqslant t_0$

are sufficient conditions to determine $x_i(t)$ for all $t \geqslant t_0$. We call n the **system order**.

3 The state, input, and **output variables** are all functions of time. Typically, we construct vector-valued functions of time for each. The so-called **state vector** $\mathbf{x}$ is actually a vector-valued function of time $\mathbf{x} : \mathbb{R} \to \mathbb{R}^n$. The ith value of $\mathbf{x}$ is a state variable denoted x_i.

4 Similarly, the so-called **input vector** $\mathbf{u}$ is actually a vector-valued function of time $\mathbf{u} : \mathbb{R} \to \mathbb{R}^r$, where r is the number of *inputs*. The ith value of $\mathbf{u}$ is an input variable denoted u_i.

5 Finally, the so-called **output vector** $\mathbf{y}$ is actually a vector-valued function of time $\mathbf{y} : \mathbb{R} \to \mathbb{R}^m$, where m is the number of *outputs*. The ith value of $\mathbf{y}$ is an output variable denoted y_i.

6 Most systems encountered in engineering practice can be modeled as state-determined. For these systems, the number of state variables n is equal to the number of **independent energy storage elements**.

7 Since to know the state vector $\mathbf{x}$ is to know everything about the state, the energy stored in each element can be determined from $\mathbf{x}$. Therefore, the time-derivative $d\mathbf{x}/dt$ describes the **power flow**.

8 The choice of state variables represented by $\mathbf{x}$ is not unique. In fact, any basis transformation yields another valid state vector. This is because, despite a vector's *components* changing when its basis is changed, a

"symmetric" change also occurs to its *basis vectors*. This means *a vector is a coordinate-independent object*, and the same goes for vector-valued functions. This is not to say that there aren't invalid choices for a state vector. There are. But if a valid state vector is given in one basis, any basis transformation yields a valid state vector.

9 One aspect of the state vector *is* invariant, however: it must always be a vector-valued function in $\mathbb{R}^n$. Our method of analysis will yield a special basis for our state vectors. Some methods yield rather unnatural state variables (e.g. the third time-derivative of the voltage across a capacitor), but ours will yield natural state variables (e.g. the voltage across a capacitor or the force through a spring).

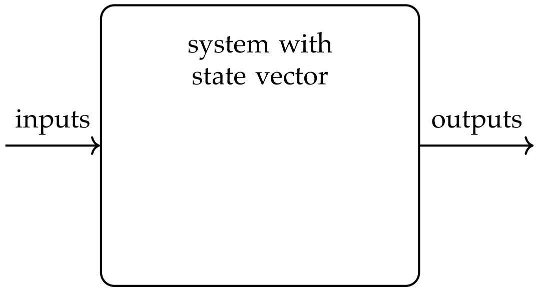

Figure svar.1: block diagram of a system with input u, state x, and output y.