

03.4 ss.nt2ss Normal tree to state-space

1 At long last, we consider an algorithm to generate a state-space model from a linear graph model. In the following, we will consider a connected graph with E edges, of which S are sources (split between through-variable sources S_T and across S_A). There are $2E - S$ unknown across- and through-variables, so that's how many equations we need. We have $E - S$ elemental equations and for the rest we will write continuity and compatibility equations. N is the number of nodes.

1. Derive $2E - S$ independent differential and algebraic equations from elemental, continuity, and compatibility equations.
 - a) Draw a **normal tree**.
 - b) Identify **primary** and **secondary variables**.
 - c) Select the **state variables** to be
across-variables on A-type branches and
through-variables on T-type links.
 - d) Define the **state vector** \mathbf{x} , **input vector** \mathbf{u} , and **output vector** \mathbf{y} .
 - e) Write an **elemental equation** for each passive element.⁴
 - f) Write a **continuity equation** for each passive branch by drawing a contour intersecting that and no other branch. Solve each for the secondary through-variable associated with that branch.⁵
 - g) Write a **compatibility equation** for each passive link by temporarily "including" it in the tree and finding the compatibility equation for the resulting loop. Solve each for the secondary across-variable associated with that link.⁶
2. Eliminate variables that are not state or input variables and their derivatives. The following procedure is recommended.

⁴There will be $E - S$ elemental equations.

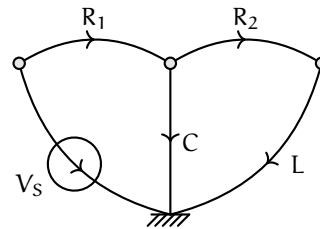
⁵There will be $N - 1 - S_A$ independent continuity equations.

⁶There will be $E - N + 1 - S_T$ independent compatibility equations.

- Eliminate all secondary variables by substitution into the elemental equations of the continuity and compatibility equations.
- Reduce the resulting set of equations to n (system order) in state and input variables, only. If not elimination, use linear algebra.
- Write the result in standard form (Equation 1a or Equation 2a).
- Express the output variables in terms of state and input variables, using any of the elemental, continuity, or compatibility equations.
- Write the result in standard form (Equation 1b or Equation 2b).

Example 03.4 ss.nt2ss-1

For the electronic system shown, find a state-space model with outputs i_L , I_s , and v_{R_2} .



re:
circuit
state-
space
model



