03.4 ss.nt2ss Normal tree to state-space

1 At long last, we consider an algorithm to generate a state-space model from a linear graph model. In the following, we will consider a connected graph with E edges, of which S are sources (split between through-variable sources S_T and across S_A). There are $2E - S$ unknown across- and through-variables, so that's how many equations we need. We have $E - S$ elemental equations and for the rest we will write continuity and compatibility equations. N is the number of nodes.

1. Derive $2E - S$ independent differential and algebraic equations from elemental, continuity, and compatibility equations.
   a) Draw a **normal tree**.
   b) Identify **primary** and **secondary variables**.
   c) Select the **state variables** to be *across*-variables on A-type branches and *through*-variables on T-type links.
   d) Define the **state vector** $\mathbf{x}$, **input vector** $\mathbf{u}$, and **output vector** $\mathbf{y}$.
   e) Write an **elemental equation** for each passive element.[4]
   f) Write a **continuity equation** for each passive branch by drawing a contour intersecting that and no other branch. Solve each for the secondary through-variable associated with that branch.[5]
   g) Write a **compatibility equation** for each passive link by temporarily "including" it in the tree and finding the compatibility equation for the resulting loop. Solve each for the secondary across-variable associated with that link.[6]
2. Eliminate variables that are not state or input variables and their derivatives. The following procedure is recommended.

[4] There will be $E - S$ elemental equations.
[5] There will be $N - 1 - S_A$ independent continuity equations.
[6] There will be $E - N + 1 - S_T$ independent compatibility equations.

a) Eliminate all secondary variables by substitution into the elemental equations of the continuity and compatibility equations.
b) Reduce the resulting set of equations to n (system order) in state and input variables, only. If not elimination, use linear algebra.
c) Write the result in standard form (Equation 1a or Equation 2a).
d) Express the output variables in terms of state and input variables, using any of the elemental, continuity, or compatibility equations.
e) Write the result in standard form (Equation 1b or Equation 2b).

Example 03.4 ss.nt2ss-1

re: circuit state-space model

For the electronic system shown, find a state-space model with outputs i_L, I_s, and v_{R_2}.