04.1 emech.trans Ideal transducers

1 **Two-port** system elements can model **transducers**—elements that transfer energy between two energy domains or change its form within an energy domain. The quintessential example, which we will consider in detail, is the **motor**, which converts electrical energy to mechanical energy. However, many other system elements can be considered transducers, and we'll consider a few in this lecture.

2 Each of the two ports has a through- and an across-variable. We use the convention that the power *into* each port ($\mathcal{P}_1$ and $\mathcal{P}_2$) is positive, which has implications for the signs of the power flow variables $\mathcal{F}_1$, $\mathcal{F}_2$, $\mathcal{V}_1$, and $\mathcal{V}_2$. For an two-port element to transfer power, we have

We define the **transformer ratio** TF to be

$$\mathrm{TF} \equiv \frac{\mathcal{V}_1}{\mathcal{V}_2} = -\frac{\mathcal{F}_2}{\mathcal{F}_1}. \tag{1}$$

Furthermore, we define the **gyrator modulus** GY to be

$$\mathrm{GY} \equiv \frac{\mathcal{V}_1}{\mathcal{F}_2} = -\frac{\mathcal{V}_2}{\mathcal{F}_1}. \tag{2}$$

3 For an **ideal transducer**—one that is linear, time-invariant, and without power loss—we have only two nontrivial solutions:[1]

$$\mathcal{V}_2 = \mathcal{V}_1/\mathrm{TF} \qquad \text{or} \qquad \mathcal{V}_2 = -\mathrm{GY}\,\mathcal{F}_1$$
$$\mathcal{F}_2 = -\mathrm{TF}\,\mathcal{F}_1 \qquad\qquad \mathcal{F}_2 = \mathcal{V}_1/\mathrm{GY}.$$

4 For a given element, if the solution with TF is a good model, we call that element a **transformer**. If the GY solution is a good model, we call it a **gyrator**.

[1] For an explanation of *why* that is the case, see Rowell **and** Wormley (1997).

Example 04.1 emech.trans-1 **re: DC motor**

Consider a DC motor with rotor radius r, number of coil turns N, background field B, and rotor length ℓ. The torque T of a DC motor is related to its coil current i by the relation

$$T = -2rNB\ell i.$$

1. Determine if DC motors are transformers or gyrators.
2. Find TF or GY.
3. Derive the relation between the voltage v and the angular velocity Ω across the motor using the assumption that it is an ideal transducer.

Example 04.1 emech.trans-2 **re: gears**

Consider two gears with radii r_1 and r_2 and number of teeth n_1 and n_2.

1. Determine the power flow variables for gears.
2. Write two independent equations relating the power flow variables.
3. Determine if gears are transformers or gyrators.
4. Find TF or GY.