

## 04.1 emech.trans Ideal transducers

- 1 **Two-port** system elements can model **transducers**—elements that transfer energy between two energy domains or change its form within an energy domain. The quintessential example, which we will consider in detail, is the **motor**, which converts electrical energy to mechanical energy. However, many other system elements can be considered transducers, and we'll consider a few in this lecture.
- 2 Each of the two ports has a through- and an across-variable. We use the convention that the power *into* each port ( $\mathcal{P}_1$  and  $\mathcal{P}_2$ ) is positive, which has implications for the signs of the power flow variables  $\mathcal{F}_1$ ,  $\mathcal{F}_2$ ,  $\mathcal{V}_1$ , and  $\mathcal{V}_2$ . For an two-port element to transfer power, we have



We define the **transformer ratio** TF to be

$$\text{TF} \equiv \frac{\mathcal{V}_1}{\mathcal{V}_2} = -\frac{\mathcal{F}_2}{\mathcal{F}_1}. \quad (1)$$

Furthermore, we define the **gyrator modulus** GY to be

$$\text{GY} \equiv \frac{\mathcal{V}_1}{\mathcal{F}_2} = -\frac{\mathcal{V}_2}{\mathcal{F}_1}. \quad (2)$$

- 3 For an **ideal transducer**—one that is linear, time-invariant, and without power loss—we have only two nontrivial solutions:<sup>1</sup>

$$\begin{array}{ccc} \mathcal{V}_2 = \mathcal{V}_1/\text{TF} & \text{or} & \mathcal{V}_2 = -\text{GY} \mathcal{F}_1 \\ \mathcal{F}_2 = -\text{TF} \mathcal{F}_1 & & \mathcal{F}_2 = \mathcal{V}_1/\text{GY}. \end{array}$$

- 4 For a given element, if the solution with TF is a good model, we call that element a **transformer**. If the GY solution is a good model, we call it a **gyrator**.

<sup>1</sup>For an explanation of *why* that is the case, see Rowell and Wormley (1997).

**Example 04.1 emech.trans-1****re: DC  
motor**

Consider a DC motor with rotor radius  $r$ , number of coil turns  $N$ , background field  $B$ , and rotor length  $\ell$ . The torque  $T$  of a DC motor is related to its coil current  $i$  by the relation

$$T = -2rNB\ell i.$$

1. Determine if DC motors are transformers or gyrators.
2. Find TF or GY.
3. Derive the relation between the voltage  $v$  and the angular velocity  $\Omega$  across the motor using the assumption that it is an ideal transducer.

**Example 04.1 emech.trans-2****re:  
gears**

Consider two gears with radii  $r_1$  and  $r_2$  and number of teeth  $n_1$  and  $n_2$ .

1. Determine the power flow variables for gears.
2. Write two independent equations relating the power flow variables.
3. Determine if gears are transformers or gyrators.
4. Find TF or GY.

