04.3 emech.dcm DC motors

1 DC motors are commonly used in mechanical engineering designs as an actuator. Products such as pumps, fans, conveyors, and robots use DC motors to convert electrical energy to mechanical (rotational) energy.

2 DC motors first emerged in the mid-19th century as the first device to produce useful mechanical work from electrical power.[3] One of fathers of the DC motor, the Benedictine priest Ányos Jedlik, invented the key facets of the motor: the **stator**, the **rotor**, and the **commutator**. Roughly speaking, for a typical brushed DC motor, current flowing through the wire windings of the stator produces a magnetic field that turns the rotor, which has windings of its own; the commutator mechanically switches the direction of current flow through the windings to yield continuous electromagnetic torque.

3 We will begin our study of DC motors with a review of a key physical phenomenon: the mechanical force on a charged particle moving in a magnetic field.

Lorentz force

4 Consider a charged particle moving through a background magnetic field. The **Lorentz force** is the (mechanical!) force on the particle, which depends on the velocity of the particle, the background magnetic field, and the background electric field. Charge flowing through a straight, stationary[4] wire with current i in a uniform background magnetic field $\mathbf{B}$ is subject to the cumulative effect of the Lorentz force on each charge. Let the straight wire's length and orientation in the B-field be described by the vector $\boldsymbol{\ell}$, which should be chosen to be in the direction of positive current flow. It can be shown that the resultant force $\mathbf{f}$ on the wire is

$$\mathbf{f} = i\boldsymbol{\ell} \times \mathbf{B} \tag{1}$$

as shown in Fig. dcm.1.

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[3] See a decent history here.

[4] The equations here assume a stationary wire. In a DC motor, the wire is moving, which creates additional effects, but the Lorentz force is still present.

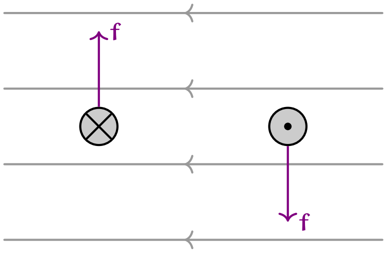


Figure dcm.1: the forces f on two wires in a magnetic field B. The wire on the left has current flowing *into* the board, that on the right has current flowing *out of* the board. The cross-product right-hand-rule applies.

5 With a curved wire, then, we could take infinitesimal sections $d\ell$ and integrate along the wire's path:

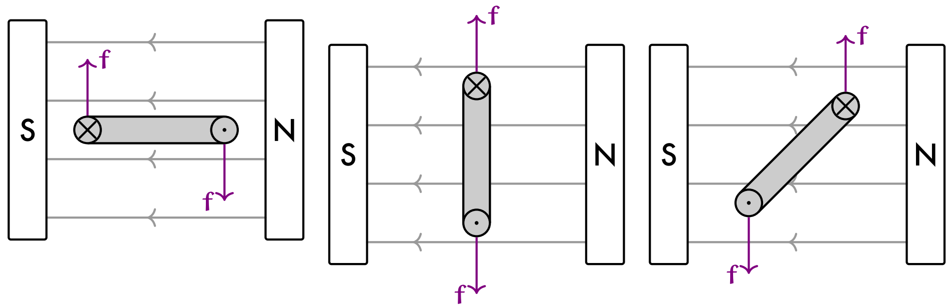
$$\mathbf{f} = i \int d\ell \times \mathbf{B}. \tag{2}$$

6 DC motors take advantage of this electromechanical phenomenon by driving current through cleverly arranged wires to generate torque on a shaft.

Permanent magnet DC motors

7 In order to take advantage of the Lorentz force, first a uniform background magnetic field $\mathbf{B}$ is required. Some DC motors, called **permanent magnet DC motors** (PMDC motors) generate this field with two stationary permanent magnets arranged as shown in Fig. dcm.2. The magnets are affixed to the "stationary" part of the motor called the **stator**.

8 Now consider a rigidly supported wire with current i passing through the field such that much of its length is perpendicular to the magnetic field. Consider the resultant forces on these perpendicular sections of wire for different wire configurations, as illustrated in Fig. dcm.2. We have torque! But note that it changes direction for different armature orientations, which will need to be addressed in a moment. Note that we can wind this wire—which we call the **armature**—multiple times around the loop to increase the torque. The rotating bit of the motor that supports the armature is called the **rotor**, which includes the shaft.



(a) Orientation A: horizontal.
(b) Orientation B: vertical.
(c) Orientation C: mixed.

Figure dcm.2: axial section view of a simple DC motor with permanent magnets.

9 The trouble is, if we connect our armature up to a circuit—which is usually located alongside the stator, i.e. *not rotating*—the wire will wrap about itself, which is **not #winning**. But we're tricky af so let's consider just cutting that wire and rigidly connecting it to a disk—called a **commutator**—with two conductive regions, one for each terminal of the armature. The commutator will rotate with the armature, but it provides smooth contacts along the perimeter of the disk.

10 We can then connect the driving circuit to these contacts via **brushes**: conductive blocks pressed against the commutator on opposite sides such that they remain in contact (conducting current) yet allow the commutator to slide easily, as shown in Fig. dcm.3. Brushes are typically made from carbon and wear out over time. This is partially mitigated by spring-loading, but eventually the brushes must be replaced, nonetheless.

11 So brushes solve the "wire wrapping" problem, but do they have an effect on the "torque flipping" issue? Yes! When the armature passes through its vertical orientation, *current reverses direction through the armature*. So whenever the perpendicular section of wire is on the right, current flows in the same direction, regardless of to which side of the armature it belongs.

12 Finally, is there a way to overcome the limitation of torque variation with different armature angles? Yes: if there are several different armature windings at different angles and correspondingly the commutator is split

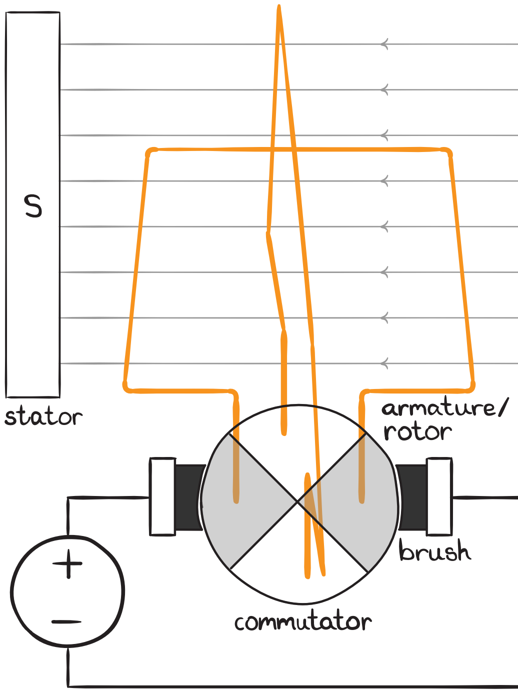


Figure dcm.3: illustration of brushes, commutator, and two armatures.

into several conductive contact pairs (one for each armature winding), a relatively continuous torque results! Real PMDC motors use this technique.

Wound stator DC motors

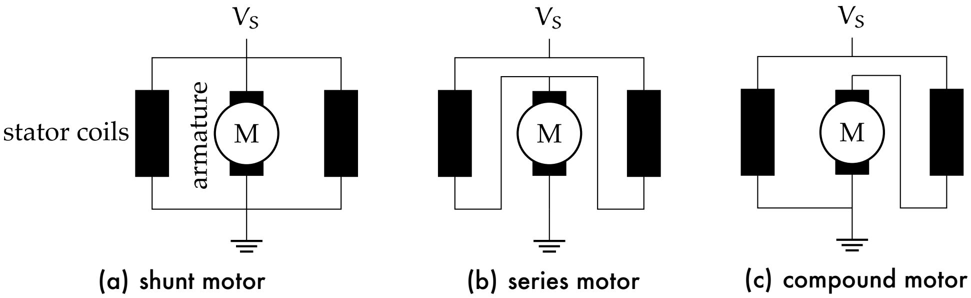
13 **Wound stator DC motors** operate very similarly to PMDC motors, but generate their background field with two stationary coils in place of the permanent magnets, above. These electromagnets require a current of their own, which is usually provided through the same circuitry that supplies the armature current (DC motors typically have only two terminals).

14 Three common configurations of the electrical connection of these are shown in Fig. dcm.4. These define the following three DC motor types.

shunt The **shunt DC motor** has its stator and rotor windings connected in parallel. These are the most common wound stator DC motors and their speeds can be easily controlled without feedback, but they have very low starting torque.

series The **series DC motor** has stator and rotor windings connected in series. These have high starting torque—so high, in fact, that it is not advisable to start these motors without a load—but their speeds are not as easily controlled without feedback.

compound The **compound DC motor** has stator and rotor windings connected in both series and parallel. These can exhibit characteristics that mix advantages and disadvantages of shunt and series DC motors.



(a) shunt motor (b) series motor (c) compound motor

Figure dcm.4: connections for shunt, series, and compound DC motors.

Brushless DC motors

15 There is yet another type of DC motor: **brushless** (BLDC). Brushless DC motors work on principles more similar to AC motors, but require complex solid-state switching that must be precisely timed. As their name implies, these motors do not require brushes. A brushless DC motor mathematical model is not presented here, but a nice introduction is given by Baldursson (2005).

16 The brushed DC motor is still widely used, despite its limitations, which include relatively frequent maintenance to replace brushes that wear out or clean/replace commutators. Other disadvantages of brushed DC motors include their relatively large size, relatively large rotor inertia, heat generated by the windings of the stator and/or rotor, and arcing that creates

electronic interference for nearby electronics. Reasons they are still widely used include that they are inexpensive (about half the cost of brushless DC motors), don't require (but often still use) complex driving circuits, are easy to model, and are easily driven at different speeds; for these reasons, an additional reason emerges: they're relatively easy to design with!

A PMDC motor model

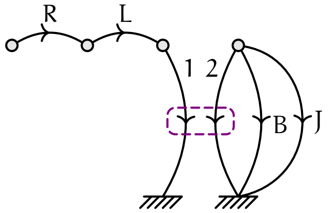


Figure dcm.5: a better brushed DC motor model.

17 We have already explored a model for a PMDC motor in Example 04.1 emech.trans-1, which yielded elemental equations

$$T_2 = -TF i_1 \quad \text{and} \tag{3}$$

$$\Omega_2 = v_1 / TF, \tag{4}$$

where TF is the motor constant. That model assumed neither armature resistance nor inductance were present—that is, it was an ideal transformer model. A linear graph of a much better model for a DC motor is shown in Figure dcm.5. This model includes a resistor R and inductor L in series with an ideal transducer. On the mechanical side, the rotor inertia J and internal bearing damping B are included. The tail ends of R and 2 should be connected to external electrical and mechanical subgraphs, respectively.

Motor constants

18 The **motor torque constant** K_t and **back-emf voltage constant** K_v are related to the transformer ratio TF derived above to characterize a brushed DC motor's response. If expressed in a set of consistent units—say, SI units—K_t and K_v have the same numerical value and are equivalent to TF. Precisely, with consistent units, $TF = K_v = K_t$.

19 However, manufacturers usually use weird units like oz–in/A and V/krpm. If they are given in anything but SI units, we recommend converting to SI for analysis.

20 Once in SI, we will have something like (for $x \in \mathbb{R}$):

21 So if we are given either K_t or K_v, the unknown constant can be found (in SI units) by converting the known constant to SI.[5]

Animations

22 There are some great animations of DC motor operating principles and construction. I've included the url of my favorite, along with some bonus animations for other important types of motors we don't have time to discuss, here.

- Brushed DC motors: `youtu.be/LAtPHANEfQo`
- Brushless DC motors: `youtu.be/bCEiOnuODac`
- AC (asynchronous) induction motors: `youtu.be/AQqyGNOP_3o`
- AC synchronous motors: `youtu.be/Vk2jDXxZIhs`
- Stepper motors: `youtu.be/eyqwLiowZiU`

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[5] One more note. When given a torque constant, the unit "oz" means "ounce-force," which is the mass in regular (mass) ounces multiplied by the gravitational acceleration g.