

04.4 emech.real Modeling a real electromechanical system

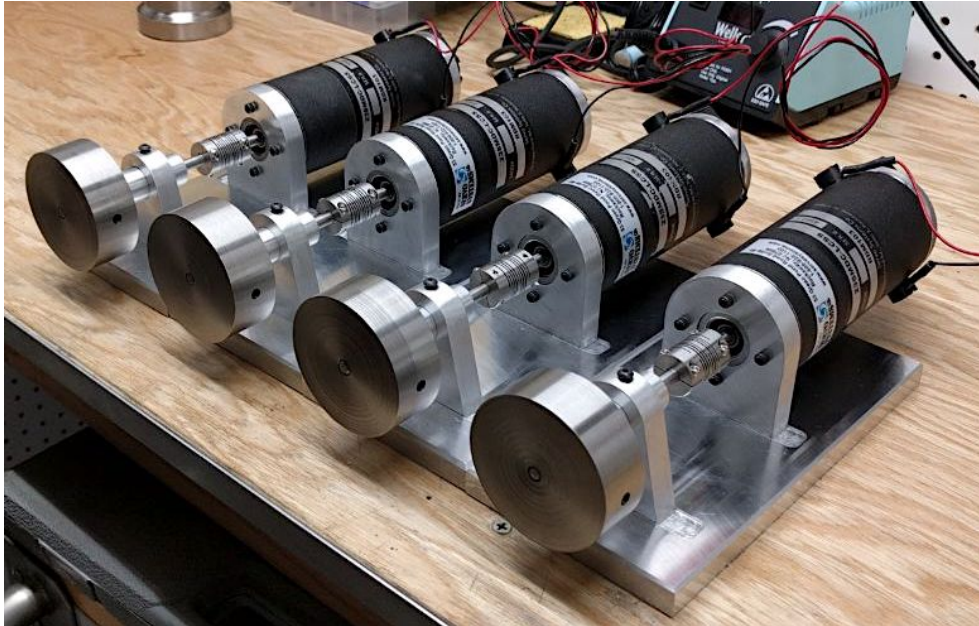


Figure real.1: electromechanical systems from the lab.

1 We now model the electromechanical systems from the laboratory, shown in [Figure real.1](#). The system includes a brushed DC motor (*Electrocraft 23SMDC-LCSS servomotor from Servo Systems*), two shafts, a shaft coupler, two bearings, and a flywheel. The motor's datasheet specifications are given in [Table real.1](#). The mechanical subsystem's inertia is dominated by the stainless steel flywheel with $J_f = 0.324 \cdot 10^{-3} \text{ kg}\cdot\text{m}^2$. The bearing damping B_b is the most difficult parameter to determine. Let's begin with the assumption that the combined bearing damping is $B_b = 20 \cdot 10^{-6} \text{ N}\cdot\text{m}/(\text{rad}/\text{s})$.

⁶Load applied at one inch from bearing.

Table real.1: datasheet specifications for the *Electrocraft 23SMDC-ICSS* servomotor from *Servo Systems*. This is the motor used in the lab.

| | parameter | specification | SI conversion |
|------------------|------------------------------------|----------------------------|---|
| general | continuous stall torque | 55 oz-in | 0.388 N-m |
| | peak torque T_{max} | 400 oz-in | 2.82 N-m |
| | max terminal voltage | 60 V _{dc} | 60 V _{dc} |
| | max operating speed Ω_{max} | 6000 rpm | 628 rad/s |
| mechanical | rotor inertia J_m | 0.008 oz-in/s ² | $56.5 \cdot 10^{-6}$ N-m/s ² |
| | damping constant B_m | 0.25 oz-in/krpm | $16.9 \cdot 10^{-6}$ N-m/(rad/s) |
| | thermal resistance | 4 C/W | 4 K/W |
| | max armature temp | 155 C | 428 K |
| | max friction torque | 3 oz-in | 0.0212 N-m |
| | max radial load ⁶ | 10 lb | 44.5 N |
| | weight (motor only) | 3.5 lb | 15.6 N |
| electrical | torque constant K_t | 13.7 oz-in/A | 0.097 N-m/A |
| | voltage constant K_v | 10.2 V/krpm | 0.097 V/(rad/s) |
| | terminal resistance | 1.6 Ω | 1.6 Ω |
| | electrical time constant | 2.6 ms | $2.6 \cdot 10^{-3}$ s |
| | mechanical time constant | 8.9 ms | $8.9 \cdot 10^{-3}$ s |
| | max continuous current | 4 A | 4 A |
| | armature inductance | 4.1 mH | $4.1 \cdot 10^{-3}$ H |
| max peak current | 34 A | 34 A | |

Linear graph model

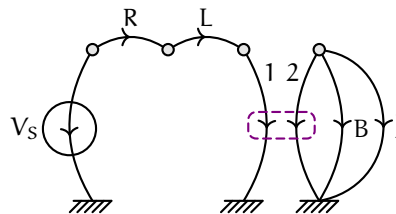


Figure real.2: a linear graph model of the electromechanical systems of [Figure real.1](#).

2 A linear graph model is in order. An ideal voltage source drives the

motor⁷—modeled as an ideal transducer with armature resistance R and inductance L , given in [Table real.1](#). The ideal transducer's rotational mechanical side (2) is connected to a moment of inertia $J = J_m + J_f = 0.381 \cdot 10^{-3} \text{ kg}\cdot\text{m}^2$, dominated by the flywheel,⁸ and damping B , which is the parallel combination of the internal motor damping of [Table real.1](#) and the bearing damping B_b , to yield $B = 26.9 \cdot 10^{-6} \text{ N}\cdot\text{m}/\text{s}^2$. We choose to ignore the flexibility of the coupler. [Problem 04.4 emech.](#) considers the same system but does not ignore the coupler's flexibility. In general, shaft couplers have significant flexibility and, depending on the application, this may require consideration in the dynamic model.

State-space model

3 The normal tree can be constructed by the procedure from [Lecture 04.2 emech.transmod](#). The voltage source V_s is first included, followed by J . Then exactly one edge of the ideal transducer must be selected, minimizing the number of T-types in the tree. We don't really have a choice, in this case, because selecting edge 2 would create a loop, so we must select edge 1. Next, R is included. No more elements can be included without creating a loop, so we are finished.

4 We are now prepared to determine variables. The state variables are across variables of A-type tree branches and through variables of T-type links—so Ω_J and i_L , and the system is second-order ($n = 2$). Clearly, the system's input is the voltage source V_s . We are interested in all the variables for the analysis in [Lecture 04.5 emech.curves](#), so we choose them all for our outputs. In summary, then, the state, input, and output vectors are:

$$\mathbf{x} = \begin{bmatrix} \Omega_J \\ i_L \end{bmatrix}, \quad \mathbf{u} = [V_s], \quad \text{and}$$

$$\mathbf{y} = [\Omega_J \quad T_J \quad v_L \quad i_L \quad \Omega_B \quad T_B \quad v_R \quad i_R \quad v_1 \quad i_1 \quad \Omega_2 \quad T_2 \quad V_s \quad I_s]^T.$$

⁷Often we can model our motor-driving source as ideal within an operating range. See [Lecture 04.7 emech.drive](#) for more details.

⁸This is the sum of the inertia of the flywheel $J_f = 0.324 \cdot 10^{-3} \text{ kg}\cdot\text{m}^2$ and the rotor $J_m = 0.0565 \cdot 10^{-3} \text{ kg}\cdot\text{m}^2$. It might be worthwhile combining this with the inertia from the shaft and coupler to obtain a more accurate value, but the difference is likely negligible.

5 Let's write some equations! Elemental are up first.

| | |
|---|---|
| J | R |
| L | 1 |
| B | 2 |

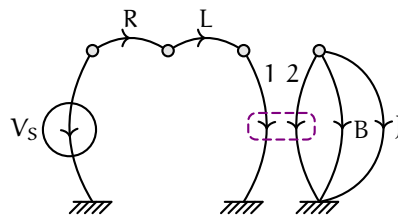


Figure real.3: the linear graph model for drawing contours.

Now, continuity and compatibility equations are developed by summing through-variables into contours. The three required contours—one for each of R, 1, and J—can be drawn on Figure real.3. The three compatibility equations—one for each of L, 2, and B—are found by “temporarily including” those links in the tree and summing across-variables around the loops created. Let's write the equations.

| branch | continuity equation | link | compatibility equation |
|--------|---------------------|------|------------------------|
| R | | L | |
| 1 | | 2 | |
| J | | B | |

6 All that remains to form the state-space model is to eliminate variables that are neither states nor inputs from the elemental, continuity, and compatibility equations. Eliminating secondary variables by substituting

the continuity and compatibility equations into the elemental equations, the following results.

$$\begin{array}{c|c} J & R \\ \hline L & 1 \\ \hline B & 2 \end{array}$$

The last four equations allow us to eliminate the remaining undesirable variables to obtain the state model in the standard form⁹

$$\frac{dx}{dt} = Ax + Bu \tag{1a}$$

$$y = Cx + Du \tag{1b}$$

where

$$A = \begin{bmatrix} -B/J & TF/J \\ -TF/L & -R/L \end{bmatrix}, \tag{1c}$$

$$B = \begin{bmatrix} 0 \\ 1/L \end{bmatrix}, \tag{1d}$$

$$C = \begin{bmatrix} 1 & -B & -TF & 0 & 1 & B & 0 & 0 & TF & 0 & 1 & 0 & 0 & 0 \\ 0 & TF & -R & 1 & 0 & 0 & R & 1 & 0 & 1 & 0 & -TF & 0 & 1 \end{bmatrix}^T, \text{ and} \tag{1e}$$

$$D = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}^T. \tag{1f}$$

⁹Here is the `rnd` file for use with StateMint (statemint.stmartin.edu) to derive the state-space model from the elemental, continuity, and compatibility equations:

```
ricopic.one/dynamic_systems/source/motor_model.rnd
```

Note that the “constraint equations” are the continuity and compatibility equations solved for primary variables.