05.1 lti.super+ Superposition, derivative, and integral properties

1 From the principle of **superposition**, **linear, time invariant** (LTI) system responses to both initial conditions and nonzero forcing can be obtained by summing the free response y_{fr} and forced response y_{fo}:

$$y(t) = y_{fr}(t) + y_{fo}(t).$$

Moreover, superposition says that any linear combination of inputs yields a corresponding linear combination of outputs. That is, we can find the response of a system to each input, separately, then linearly combine (scale and sum) the results according to the original linear combination. That is, for inputs u_1 and u_2 and constants $a_1, a_2 \in \mathbb{R}$, a forcing function

would yield output

where y_1 and y_2 are the outputs for inputs u_1 and u_2, respectively.

2 This powerful principle allows us to construct solutions to complex forcing functions by decomposing the problem. It also allows us to make extensive use of existing solutions to common inputs.

3 There are two more LTI system properties worth noting here. Let a system have input u_1 and corresponding output y_1. If the system is then given input $u_2(t) = \dot{u}_1(t)$, the corresponding output is

Similarly, if the same system is then given input $u_3(t) = \int_0^t u_1(\tau)d\tau$, the corresponding output is

These are sometimes called the **derivative** and **integral properties** of LTI systems.