05.4 lti.ghost When gravity ghosts you

1 You're familiar with experience. Just when you think you're getting along so well with a "vertically" oriented translational mechanical system, gravity stops answering your texts. In this lecture, I'll try to explain this common experience: it seems that sometimes the force of gravity "matters," and other times it does not. Is gravity really Hamletic, an ambivalent vixen, or is there some way to understand this phenomenon?

2 Consider the following example contrived to shed some light.

Example 05.4 lti.ghost-1

re: state-space model of a harmonic oscillator

3 Often when considering a spring k, we focus on the *velocity* v_k *across it*, i.e. the time-derivative of the *displacement* x_k. We effectively differentiate-away the constant unstretched length L from x_k; we can think of

$$x_k - L \quad (1)$$

as the "stretch" of the spring. In this exercise, we will attend closely to the details of this stretching.

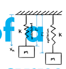

Figure ghost.1: a mechanical harmonic oscillator with spring k unstretched (left) and stretched (right) to its static equilibrium length.

4 Consider the mechanical harmonic oscillator shown in Fig. ghost.1. Derive a single input-output ODE for the system in terms of x_k, the total displacement across the spring. Let the constant $\widetilde{L}$ be the stretched length of the spring when the system is in static equilibrium. Solve for $\widetilde{L}$ in terms of the system parameters. Show that when we change ODE dependent variable from x_k to

$$\widetilde{x}_k = x_k - \widetilde{L}, \quad (2)$$

the *displacement from equilibrium*, gravity ghosts us!

5 For this example, there is a much shorter way to deriving the system ODE than our usual approach, and we will use it here: the traditional free-body diagram application of Newton's laws, shown in Fig. ghost.2. Applying

Newton's second law,

where the forces are

(when $x_k > 0$, $f_k < 0$)

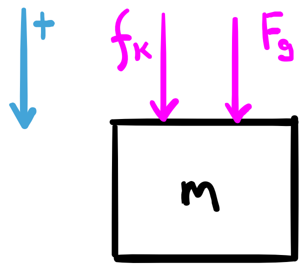

Figure ghost.2: a free-body diagram of the mass.

Substituting in the forces,

6 Equilibrium implies $\ddot{x}_k = 0$ and $x_k = \widetilde{L}$, therefore

7 Changing variables *à la* Eq. 2 in the ODE yields

Alas, poor ghost!

8 We have seen now that the gravitational ghosting occurs when we change variables such that the displacement is relative to an *equilibrium* in the gravitational field. It is simply a change of **datum** or *reference* position of the displacement that cancels out the gravitational term—ay, there's the rub! We call this the **equilibrium requirement**.

9 For this reason, those performing such analyses, with a flourish of the hand, declare vertical displacements relative to equilibrium and poof—gravity disappears without explicit justification, for the details make cowards of us all.

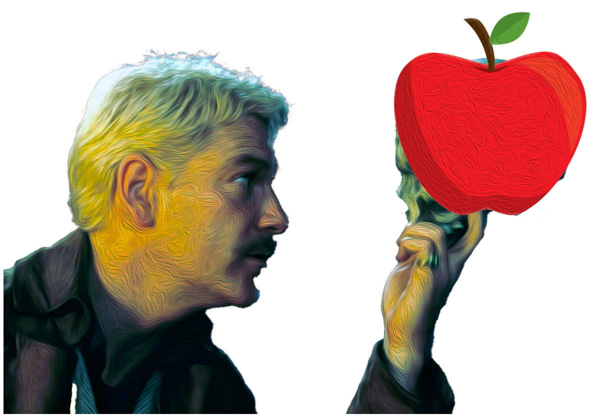

Figure ghost.3: to ghost or not to ghost, that is the question.

10 But there are situations in which this would be a fatal error: those for which there is ____________________! For instance, consider if the mass in our previous example was suspended from a damper instead of a spring: in this case, no equilibrium exists! Without going through the details or at least recalling the equilibrium requirement, it can be easy to fool oneself into wrongly dismissing gravity.

11 *Remember me,*
Ghost-would-be,
For I am thy father's spirit,
If gravity'd,
With thee flee,
Th'equilibrium requirement.