05.5 lti.exe Exercises for Chapter 05 lti

Exercise 05.1 oil

A certain sensor used to measure displacement over time t is tested several times with input displacement $u_1(t)$ and a certain function $y_1(t)$ is estimated to properly characterize the corresponding voltage output. Assuming the sensor is linear and time-invariant, what would we expect the output sensor voltage $y_2(t)$ to be when the following input is applied?

_____/ 10 p.

$$u_2(t) = 3\,\dot{u}_1(t) - 5\,u_1(t) + \int_0^t 6\,u_1(\tau)\,d\tau \tag{1}$$

Exercise 05.2 water

A system with input $u(t)$ and output $y(t)$ has the governing dynamical equation

_____/ 15 p.

$$2\,\ddot{y} + 12\,\dot{y} + 50\,y = -10\dot{u} + 4u. \tag{2}$$

a. What is the equilibrium $y(t)$ when $u(t) = 6$?
b. Demonstrate the stability, marginal stability, or instability of the system.

Exercise 05.3 timmychalamet

The free response of a linear system with a given set of initial conditions is y_{fr}. The forced response of the system to input u_1 is y_{fo_1}. The forced response of the system to input u_2 is y_{fo_2}. What is the (specific) response of the system to the same set of initial conditions when $u_1(t) + u_2(t)$ is also applied? Express your answer in terms of y_{fr}, y_{fo_1}, and y_{fo_2}.

_____/ 10 p.

_____/ 15 p.

Exercise 05.4 flopugh

Consider a linear system with state-space model matrices

$$A = \begin{bmatrix} -4 & 11 \\ 3 & -12 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 \end{bmatrix}. \tag{3}$$

For this system, respond to the following questions and imperatives.

1. What is the equilibrium state $\overline{x}$ for input $u(t) = 0$?
2. Find the corresponding input-output ODE for the system.
3. Demonstrate the asymptotic stability, marginal stability, or instability of the system from the ODE.

06 trans

1 In this chapter, we explore the qualities of transient response—the response of the system in the interval during which initial conditions dominate.

2 We focus on characterizing first- and second-order linear systems; not because they're easiest (they are), but because nonlinear systems can be **linearized** about an **operating point** and because higher-order linear system responses are just *sums of first- and second-order responses*, making "everything look first- and second-order." Well, many things, at least.

3 In this chapter, we primarily consider systems represented by **single-input, single-output (SISO)** ordinary differential equations (also called io ODEs)—with time t, *output* y, *input* u, **forcing function** f, constant coefficients a_i, b_j, order n, and $m \leqslant n$ for $n \in \mathbb{N}_0$—of the form

$$\frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_1 \frac{dy}{dt} + a_0 y = f, \text{ where} \tag{1a}$$

$$f \equiv b_m \frac{d^m u}{dt^m} + b_{m-1}\frac{d^{m-1} u}{dt^{m-1}} + \cdots + b_1 \frac{du}{dt} + b_0 u. \tag{1b}$$

Note that the forcing function f is related to but distinct from the input u. This terminology proves rather important.