06.1 trans.char Characteristic transient responses

1 A system's **characteristic responses** are responses to specific forcing functions—called the **singularity functions**. The reasons these are "characteristic" are:

1. the singularity functions model commonly interesting system inputs (e.g. a sudden change in the input), and so they can be said to *characterize inputs*, and
2. the ways in which the system responds to these specific functions reveal aspects of the system (e.g. how quickly it responds), so these responses can be said to *characterize systems*.

2 Now, one may object that Equation 1b shows that a forcing function needn't look anything like an input due to its being composed of a sum of scaled copies of the input and its derivatives. Yes, but given two key properties of linear, time-invariant (LTI) systems—**superposition** and the **differentiation** property—, knowing a system's response y_1 to a forcing function f_1 allows us to construct its response to that input (that is, y_2 for input $u_2 = f_1$) as

I know.

3 There are three singularity functions, which are now defined as piecewise functions of time t.

4 First, the **unit impulse** or **Dirac delta** function[1] δ is defined as zero everywhere except at $t = 0$, when it is undefined, and has unity as its integral over all time. When δ is scaled (e.g. 5δ), its integral scales by the same factor. This strange little beast models a sudden "spike" in the input.

[1]Technically, δ is a distribution, not a function, but we use the common, confusing, comfortably couched terminology.

5 Second, the **unit step** function u_s is defined as zero for $t \leqslant 0$ and unity for $t > 0$. It models a sudden change in the input. Scaling it scales the amount of change. Often, this is considered to be the gold-standard for characterizing the transient response of a system.

6 Third, the **unit ramp** function u_r is defined as zero for $t \leqslant 0$ and t for $t > 0$—that is, it is linearly increasing with unity slope. It models a steadily increasing input and is probably the least useful of the singularity functions. Scaling it scales the rate of steady change.