06.4 trans.exe Exercises for Chapter 06 trans

Exercise 06.1 truman

Consider the i/o ODE with independent variable t and dependent variable y: ______/ 25 p.

$$7\dot{y} + y = \dot{u} - 5u$$

with input

$$u(t) = u_r$$

the unit ramp function.

a. What is the time constant τ?
b. Find the *characteristic* response y_r of the system to the unit ramp input. Stongly consider using Table firsto.1.
c. What is the *forced* response y_{fo} to the same input?
d. What is the free response of the y_{fr} to initial condition $y(0) = 8$?
e. What is the total response y_t when both the input u and initial condition $y(0)$ are applied simultaneously?

Exercise 06.2 mogul

Consider the i/o ODE with independent variable t and dependent variable y:

$$\ddot{y} + 5\dot{y} + 25y = 2\dot{u} + 3u$$

with input

$$u(t) = u_s$$

the unit step function.

a. What are the natural frequency ω_n and damping ratio ζ?

b. Find the *characteristic* response of the system to the unit step input. Stongly consider using Table secondo.1.
c. What is the *forced* response to the unit step input?

Exercise 06.3 kibble

Consider the input-output ODE with independent variable t, dependent variable (output) $y(t)$, and input $u(t)$: ______/ 20 p.

$$\dot{y} + 3y = 2\dot{u} + u.$$

a. What is the time constant τ?
b. Find the *characteristic* response y_s of the system to the unit step input $u(t) = u_s(t)$. Stongly consider using Table firsto.1.
c. What is the *forced* response y_{fo} to the input $u(t) = 3u_s(t)$?
d. What is the *free* response of the y_{fr} to initial condition $y(0) = -4$?
e. What is the total response y_t when both the input u from Item c. and initial condition $y(0)$ are applied simultaneously?

Exercise 06.4 biology

Consider a system with the following input-output ODE with independent variable t, dependent variable (output) $y(t)$, and input $u(t)$: ______/ 30 p.

$$\ddot{y} + 5\dot{y} + 25y = \dot{u} + 7u$$

a. What are the natural frequency ω_n and damping ratio ζ?
b. Find the *characteristic* response y_δ of the system to the unit impulse forcing $f(t) = \delta(t)$. *Hint:* Stongly consider using Table secondo.1.
c. What is the *forced* response y_{fo} to the input $u(t) = \delta(t)$?
d. What is the *free* response y_{fr} to initial condition $y(0) = 11$?
e. What is the total response y_t when both the input u from Item c. and initial condition from Item d. are applied simultaneously?
f. For a constant input $u(t) = \overline{u}$, what is the equilibrium output $y(t) = \overline{y}$?
g. Demonstrate the stability, marginal stability, or instability of the system.

07 ssresp

1 Recall that, for a state-space model, the state $\mathbf{x}$, input $\mathbf{u}$, and output $\mathbf{y}$ vectors interact through two equations:

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t) \tag{1a}$$

$$\mathbf{y} = \mathbf{g}(\mathbf{x}, \mathbf{u}, t) \tag{1b}$$

where $\mathbf{f}$ and $\mathbf{g}$ are vector-valued functions that depend on the system. Together, they comprise what is called a **state-space model** of a system.

2 In accordance with the definition of a state-determined system, given an initial condition $\mathbf{x}(t_0)$ and input $\mathbf{u}$, the state $\mathbf{x}$ is determined for all $t \geqslant t_0$. Determining the state response requires the solution—analytic or numerical—of the vector differential equation Eq. 1a.

3 The second equation (1b) is *algebraic*. It expresses how the output $\mathbf{y}$ can be constructed from the state $\mathbf{x}$ and input $\mathbf{u}$. This means we must first solve the state equation (1a) for $\mathbf{x}$, then the output $\mathbf{y}$ is given by Eq. 1b.

4 Just because we know that, for a state-determined system, there *exists* a solution to Eq. 1a, doesn't mean we know how to find it. In general, $\mathbf{f} : \mathbb{R}^n \times \mathbb{R}^r \times \mathbb{R} \to \mathbb{R}^n$ and $\mathbf{g} : \mathbb{R}^n \times \mathbb{R}^r \times \mathbb{R} \to \mathbb{R}^m$ can be nonlinear functions.[1] We don't know how to solve most nonlinear state equations analytically. An additional complication can arise when, in addition to states and inputs, system parameters are themselves time-varying (note the explicit time t argument of $\mathbf{f}$ and $\mathbf{g}$). Fortunately, often a linear, time-invariant (LTI) model is sufficient.

[1]Technically, since $\mathbf{x}$ and $\mathbf{u}$ are themselves functions, $\mathbf{f}$ and $\mathbf{g}$ are *functionals*.

5 Recall that an LTI state-space model is of the form

$$\frac{d\mathbf{x}}{dt} = A\mathbf{x} + B\mathbf{u} \tag{2a}$$

$$\mathbf{y} = C\mathbf{x} + D\mathbf{u}, \tag{2b}$$

where A, B, C, and D are constant matrices containing system lumped-parameters such as mass or inductance. See Chapter 03 ss for details on the derivation of such models.

6 In this chapter, we learn to solve Eq. 2a for the state response and substitute the result into Eq. 2b for the output response. First, we learn an analytic solution technique. Afterward, we learn simple software tools for numerical solution techniques.