# 07.2 ssresp.eig Linear algebraic eigenproblem

1 The linear algebraic **eigenproblem** can be simply stated. For  $n \times n$  real matrix A,  $n \times 1$  complex vector m, and  $\lambda \in \mathbb{C}$ , m is defined as an **eigenvector** of A if and only if it is nonzero and

$$Am = \lambda m \tag{1}$$

for some  $\lambda$ , which is called the corresponding **eigenvalue**. That is, **m** is an eigenvector of A if its linear transformation by A is equivalent to its scaling; i.e. an eigenvector of A is a vector of which A changes the length, but not the direction.

2 Since a matrix can have several eigenvectors and corresponding eigenvalues, we typically index them with a subscript; e.g.  $m_i$  pairs with  $\lambda_i$ .

#### Solving for eigenvalues

Eq. 1 can be rearranged:

$$(\lambda \mathbf{I} - \mathbf{A})\mathbf{m} = \mathbf{0}.$$

For a nontrivial solution for m,

$$\det(\lambda \mathbf{I} - \mathbf{A}) = \mathbf{0},\tag{3}$$

which has as its left-hand-side a polynomial in  $\lambda$  and is called the **characteristic equation**. We define **eigenvalues** to be the roots of the characteristic equation.

## Box 07 ssresp.1 eigenvalues and roots of the characteristic equation

If A is taken to be the linear state-space representation A, and the statespace model is converted to an input-output differential equation, the resulting ODE's "characteristic equation" would be identical to this matrix characteristic equation. Therefore, everything we already understand about the roots of the "characteristic equation" of an i/o ODE—especially that they govern the transient response and stability of a system—holds for a system's A-matrix eigenvalues.

3 Here we consider only the case of n distinct eigenvalues. For eigenvalues of (algebraic) multiplicity greater than one (i.e. repeated roots), see the discussion of Appendix 02.1 adv.eig.

### Solving for eigenvectors

4 Each eigenvalue  $\lambda_i$  has a corresponding eigenvector  $m_i$ . Substituting each  $\lambda_i$  into Eq. 2, one can solve for a corresponding eigenvector. It's important to note that an eigenvector is unique within a scaling factor. That is, if  $m_i$  is an eigenvector corresponding to  $\lambda_i$ , so is  $3m_i$ .<sup>3</sup>

### Example 07.2 ssresp.eig-1

Let

A =	2	-4]
	_1	-1

Find the eigenvalues and eigenvectors of A.

re: eigenproblem for a 2 × 2 matrix

<sup>&</sup>lt;sup>3</sup>Also of note is that  $\lambda_i$  and  $m_i$  can be complex.

5 Several computational software packages can easily solve for eigenvalues and eigenvectors. See Lec. 07.3 ssresp.eigcomp for instruction for doing so in Matlab and Python.