07.3 ssresp.eigcomp Computing eigendecompositions

1 Computing eigendecompositions is rather straightforward with a numerical or symbolic computing tool such as those available in Matlab or Python. The following sections show how to use Matlab and Python to compute numerical and symbolic eigendecompositions.

Matlab eigendecompositions

Matlab numerical eigendecompositions

Consider the following matrix A.

```
A = [ ...
  -3, 5, 9; ...
  0, 2, -10; ...
  5, 0, -4 ...
];
```

What are its eigenvalues and eigenvectors? Let's use the MATLAB function eig. From the documentation:

> *[V,D] = EIG(A) produces a diagonal matrix D of eigenvalues and a full matrix V whose columns are the corresponding eigenvectors so that A*V = V*D.*

Let's try it.

```
[Ve,De] = eig(A);
disp(vpa(Ve,3))
```

```
[ -0.769,    0.122 - 0.537i,    0.122 + 0.537i]
[  0.381,             0.767,             0.767]
[  0.514, - 0.0953 - 0.316i, - 0.0953 + 0.316i]
```

The eigenvalues are on the diagonal of De.

```
disp(diag(De))
```

```
   -11.487 +         0i
    3.2433 +     4.122i
    3.2433 -     4.122i
```

The eigenevectors are normalized to have unit length.

```
disp(norm(Ve(:,3))) % for instance
```

```
     1
```

Matlab symbolic eigendecompositions

Sometimes symbolic parameters in a matrix require symbolic eigendecomposition. In Matlab, this requires the `symbolic` toolbox. First, declare symbolic variables.

```
syms a b c
```

Now form a symbolic matrix.

```
A = [ ...
  a,b; ...
  0,c; ...
]
```

```
A =
[ a, b]
[ 0, c]
```

The function `eig` is overloaded and if `A` is symbolic, the symbolic routine is called, which has a syntax similar to the numerical version above.

```
[Ve_sym,De_sym] = eig(A)
```

```
Ve_sym =
[ 1, -b/(a - c)]
[ 0,          1]
De_sym =
[ a, 0]
[ 0, c]
```

Again, the eigenvalues are on the diagonal of the eigenvalue matrix.

```
disp(diag(De_sym))
```

```
 a
 c
```

Python eigendecompositions

Python numerical eigendecompositions

In Python, we first need to load the appropriate packages.

```
import numpy as np # for numerics
from numpy import linalg as la # for eig
from IPython.display import display, Markdown, Latex # prty
np.set_printoptions(precision=3) # for pretty
```

Consider the same numerical `A` matrix from the section above. Create it as a `numpy.array` object.

```
A = np.array(
  [
    [-3, 5, 9],
    [0, 2, -10],
    [5, 0, -4],
  ]
)
```

The `numpy.linalg` module (loaded as `la`) gives us access to the `eig` function.

```
e_vals,e_vecs = la.eig(A)
print(f'e-vals: {e_vals}')
print(f'modal matrix:\n {e_vecs}')
```

```
e-vals: [-11.487+0.j      3.243+4.122j   3.243-4.122j]
modal matrix:
 [[-0.769+0.j      0.122-0.537j  0.122+0.537j]
 [ 0.381+0.j      0.767+0.j      0.767-0.j    ]
 [ 0.514+0.j     -0.095-0.316j -0.095+0.316j]]
```

Note that the eigenvalues are returned as a one-dimensional array, not along the diagonal of a matrix as with Matlab.

```
print(f"the third eigenvalue is {e_vals[2]:.3e}")
```

```
the third eigenvalue is 3.243e+00-4.122e+00j
```

Python symbolic eigendecompositions

We use the `sympy` package for symbolics.

```
import sympy as sp
```

Declare symbolic variables.

```
sp.var('a b c')
```

```
(a, b, c)
```

Define a symbolic matrix `A`.

```
A = sp.Matrix([
  [a,b],
  [0,c]
])
display(A)
```

$$\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$$

The `sympy.Matrix` class has methods `eigenvals` and `eigenvects`. Let's consider them in turn.

```
A.eigenvals()
```

```
{a: 1, c: 1}
```

What is returned is a `dictionary` with our eigenvalues as its keys and the *multiplicity* (how many) of each eigenvalue as its corresponding value.
The `eigenvects` method returns even more complexly structured results.

```
A.eigenvects()
```

```
[(a, 1, [Matrix([
   [1],
   [0]])]), (c, 1, [Matrix([
   [-b/(a - c)],
   [         1]])])]
```

This is a `list` of `tuples` with structure as follows.

```
(<eigenvalue>,<multiplicity>,<eigenvector>)
```

Each eigenvector is given as a list of symbolic matrices.
Extracting the second eigenvector can be achieved as follows.

```
A.eigenvects()[1][2][0]
```

$$\begin{bmatrix} -\frac{b}{a-c} \\ 1 \end{bmatrix}$$