07.8 ssresp.exe Exercises for Chapter 07 ssresp

Exercise 07.1 larry

Let a system have the following state and output equation matrices:

$$A = \begin{bmatrix} -3 & 0 \\ 1 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \end{bmatrix}.$$

For this system, answer the following imperatives.

a. Find the *eigenvalue matrix* Λ and comment on the stability of the system (justify your comment). Use the convention that $\lambda_1 \geqslant \lambda_2$ and order Λ accordingly.
b. Find the eigenvectors and the *modal matrix* M.
c. Find the *state transition matrix* $\Phi(t)$. Hint: first find the "diagonalized" state transition matrix $\Phi'(t)$.
d. Using the state transition matrix, find the *output free response* for initial condition

$$x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Exercise 07.2 mo

Let a system have the following state and output equation matrices:

$$A = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \end{bmatrix}.$$

For this system, answer the following imperatives.

a. Find the *eigenvalue matrix* Λ and comment on the stability of the system (justify your comment). Use the convention that $\lambda_1 \leqslant \lambda_2$ and order Λ accordingly.
b. Find the eigenvectors and the *modal matrix* M.

c. Find the *state transition matrix* $\Phi(t)$. Hint: first find the "diagonalized" state transition matrix $\Phi'(t)$.
d. Using the state transition matrix, find the *output homogeneous solution* for initial condition

$$x(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Exercise 07.3 curly

Use a computer for this exercise. Let a system have the following state A-matrix:

$$A = \begin{bmatrix} -2 & 2 & 0 \\ -1 & -2 & 2 \\ 0 & -1 & -2 \end{bmatrix}.$$

For this system, answer the following imperatives.

a. Find the *eigenvalue matrix* Λ and *modal matrix* M.
b. Comment on the stability of the system (justify your comment).
c. Find the *diagonalized state transition matrix* $\Phi'(t)$. Be sure to print the expression. Furthermore, find the *state transition matrix* $\Phi(t)$.
d. Using the state transition matrix, find the *state free response* for initial condition

$$x(0) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Do *not* print this expression.
e. Plot the free response found above for $t \in [0, 4]$ seconds.

______/
25 p.

Exercise 07.4 lonely

Use a computer for this exercise. Let a system have the following state and output equation matrices:

$$A = \begin{bmatrix} -1 & 0 & 8 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 2 \\ 3 & 0 \\ 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} \qquad D = \begin{bmatrix} 0 & 0 \end{bmatrix}.$$

For this system, answer the following imperatives.

a. Find the *eigenvalue matrix* Λ and comment on the stability of the system (justify your comment). Use the convention that $\lambda_1 \geqslant \lambda_2 \geqslant \lambda_3$ and order Λ accordingly.
b. Find the eigenvectors and the *modal matrix* M.
c. Find the *state transition matrix* $\Phi(t)$. Hint: first find the "diagonalized" state transition matrix $\Phi'(t)$.
d. Let the input be

$$\mathbf{u}(t) = \begin{bmatrix} 4 \\ \sin(2\pi t) \end{bmatrix}.$$

Solve for the forced *state* response $\mathbf{x}_{fo}(t)$. Express it simply—it's not that bad.
e. Solve for the forced *output* response $y_{fo}(t)$. Express it simply—it's not that bad.
f. Plot $y_{fo}(t)$ for $t \in [0, 7]$ sec.

Exercise 07.5 artemis

Use a computer for this exercise. Let a system have the following state and output equation matrices: ______/ 30 p.

$$A = \begin{bmatrix} -5 & 6 \\ 1 & -10 \end{bmatrix}, \qquad B = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & 1 \end{bmatrix}, \qquad D = \begin{bmatrix} 0 \end{bmatrix}.$$

For this system, answer the following imperatives.

a. Find the *eigenvalue matrix* Λ and comment on the stability of the system (justify your comment). Use the convention that $\lambda_1 \geqslant \lambda_2$ and order Λ accordingly.
b. Find the eigenvectors and the *modal matrix* M.
c. Find the *state transition matrix* $\Phi(t)$. Hint: first find the "diagonalized" state transition matrix $\Phi'(t)$.
d. Let the input be

$$\mathbf{u}(t) = \begin{bmatrix} \delta(t) \end{bmatrix},$$

where δ is the Dirac delta impulse function.[10] Solve for the forced *state* response $\mathbf{x}_{fo}(t)$. Express it simply.[11]
e. Solve for the forced *output* response $\mathbf{y}_{fo}(t)$. Express it simply.
f. Plot $\mathbf{y}_{fo}(t)$ for $t \in [0, 3]$ sec.

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[10]Matlab's `dirac` can be used, symbolically. Symbolic integration yield a result with the `heaviside` function. With the setting `sympref('HeavisideAtOrigin',0)` this is equivalent with our definition of the unit step function u_s.

[11]Matlab's `simplify` function may need some help. Use the 'assume' function.

Part III

Modeling other systems

08 thermoflu

Lumped-pa

1 We now consider the **lumped-parameter modeling** of **fluid systems** and **thermal systems**. The linear graph-based, state-space modeling techniques of Chapters 02 graphs to 04 emech are called back up to service for this purpose. Recall that this method defines several types of discrete elements in an energy domain—in Chapters 02 graphs and 03 ss, the electrical and mechanical energy domains. Also recall from Chapter 04 emech that energy transducing elements allow energy to flow among domains. In this chapter, we introduce fluid and thermal energy domains and discrete and transducing elements associated therewith.

2 The analogs between the mechanical and electrical systems from Chapter 02 graphs are expanded to include fluid and thermal systems. This generalization allows us to include, in addition to electromechanical systems, inter-domain systems including electrical, mechanical, fluid, and thermal systems.

3 This chapter begins by defining discrete lumped-parameter elements for fluid and thermal systems. We then categorize these into energy source, energy storage (A-type and T-type), and energy dissapative (D-type) elements, allowing us to immediately construct linear graphs and normal trees in the manner of Chapter 02 graphs. Then we can directly apply the methods of Chapter 03 ss to construct state-space models of systems that include fluid and thermal elements.