

07.8 ssresp.exe Exercises for Chapter 07 ssresp

Exercise 07.1 larry

Let a system have the following state and output equation matrices:

$$A = \begin{bmatrix} -3 & 0 \\ 1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 \end{bmatrix}.$$

For this system, answer the following imperatives.

- Find the *eigenvalue matrix* Λ and comment on the stability of the system (justify your comment). Use the convention that $\lambda_1 \geq \lambda_2$ and order Λ accordingly.
- Find the eigenvectors and the *modal matrix* M .
- Find the *state transition matrix* $\Phi(t)$. Hint: first find the “diagonalized” state transition matrix $\Phi'(t)$.
- Using the state transition matrix, find the *output free response* for initial condition

$$\mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Exercise 07.2 mo

Let a system have the following state and output equation matrices:

$$A = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 \end{bmatrix}.$$

For this system, answer the following imperatives.

- Find the *eigenvalue matrix* Λ and comment on the stability of the system (justify your comment). Use the convention that $\lambda_1 \leq \lambda_2$ and order Λ accordingly.
- Find the eigenvectors and the *modal matrix* M .

- c. Find the *state transition matrix* $\Phi(t)$. Hint: first find the “diagonalized” state transition matrix $\Phi'(t)$.
- d. Using the state transition matrix, find the *output homogeneous solution* for initial condition

$$\mathbf{x}(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Exercise 07.3 curly

Use a computer for this exercise. Let a system have the following state A-matrix:

$$A = \begin{bmatrix} -2 & 2 & 0 \\ -1 & -2 & 2 \\ 0 & -1 & -2 \end{bmatrix}.$$

For this system, answer the following imperatives.

- a. Find the *eigenvalue matrix* Λ and *modal matrix* M .
- b. Comment on the stability of the system (justify your comment).
- c. Find the *diagonalized state transition matrix* $\Phi'(t)$. Be sure to print the expression. Furthermore, find the *state transition matrix* $\Phi(t)$.
- d. Using the state transition matrix, find the *state free response* for initial condition

$$\mathbf{x}(0) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Do *not* print this expression.

- e. Plot the free response found above for $t \in [0, 4]$ seconds.

Exercise 07.4 lonely

Use a computer for this exercise. Let a system have the following state and output equation matrices:

$$A = \begin{bmatrix} -1 & 0 & 8 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 2 \\ 3 & 0 \\ 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 0 \end{bmatrix}.$$

For this system, answer the following imperatives.

- Find the *eigenvalue matrix* Λ and comment on the stability of the system (justify your comment). Use the convention that $\lambda_1 \geq \lambda_2 \geq \lambda_3$ and order Λ accordingly.
- Find the eigenvectors and the *modal matrix* M .
- Find the *state transition matrix* $\Phi(t)$. Hint: first find the “diagonalized” state transition matrix $\Phi'(t)$.
- Let the input be

$$\mathbf{u}(t) = \begin{bmatrix} 4 \\ \sin(2\pi t) \end{bmatrix}.$$

Solve for the forced *state* response $\mathbf{x}_{fo}(t)$. Express it simply—it’s not that bad.

- Solve for the forced *output* response $\mathbf{y}_{fo}(t)$. Express it simply—it’s not that bad.
- Plot $\mathbf{y}_{fo}(t)$ for $t \in [0, 7]$ sec.

Exercise 07.5 artemis

Use a computer for this exercise. Let a system have the following state and output equation matrices:

$$A = \begin{bmatrix} -5 & 6 \\ 1 & -10 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \end{bmatrix}.$$

For this system, answer the following imperatives.

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- Find the *eigenvalue matrix* Λ and comment on the stability of the system (justify your comment). Use the convention that $\lambda_1 \geq \lambda_2$ and order Λ accordingly.
- Find the eigenvectors and the *modal matrix* M .
- Find the *state transition matrix* $\Phi(t)$. Hint: first find the “diagonalized” state transition matrix $\Phi'(t)$.
- Let the input be

$$\mathbf{u}(t) = \begin{bmatrix} \delta(t) \end{bmatrix},$$

where δ is the Dirac delta impulse function.¹⁰ Solve for the forced *state* response $\mathbf{x}_{fo}(t)$. Express it simply.¹¹

- Solve for the forced *output* response $\mathbf{y}_{fo}(t)$. Express it simply.
- Plot $\mathbf{y}_{fo}(t)$ for $t \in [0, 3]$ sec.

Exercise 07.6 level

Use a computer for this exercise. Let a system have the following state and output equation matrices:

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$$A = \begin{bmatrix} -4 & -3 & 0 \\ 0 & -8 & 4 \\ 0 & 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \end{bmatrix}.$$

For this system, answer the following imperatives.

- Find the *eigenvalue matrix* Λ and comment on the stability of the system (justify your comment).
- Find the eigenvectors and the *modal matrix* M .
- Find the *state transition matrix* $\Phi(t)$. Hint: first find the “diagonalized” state transition matrix $\Phi'(t)$.

¹⁰Matlab's `dirac` can be used, symbolically. Symbolic integration yield a result with the `heaviside` function. With the setting `sympref('HeavisideAtOrigin', 0)` this is equivalent with our definition of the unit step function u_s .

¹¹Matlab's `simplify` function may need some help. Use the 'assume' function.

- d. Let the input be

$$\mathbf{u}(t) = \begin{bmatrix} u_s(t) \end{bmatrix},$$

where u_s is the unit step function.¹² Solve for the forced *state* response $\mathbf{x}_{fo}(t)$. Express it simply.

- e. Solve for the forced *output* response $\mathbf{y}_{fo}(t)$. Express it simply.

- f. Plot $\mathbf{y}_{fo}(t)$ for $t \in [0, 5]$ sec.

¹²In Python, we can define a symbolic unit step function using the `sympy.Heaviside()` function. Alternatively, we can set $u_s(t)$ equal to 1 for integration over the interval $[0, t]$.

Part III

Modeling other systems

08 thermoflu

Lumped-par

1 We now consider the **lumped-parameter modeling** of **fluid systems** and **thermal systems**. The linear graph-based, state-space modeling techniques of [Chapters 02 graphs](#) to [04 emech](#) are called back up to service for this purpose. Recall that this method defines several types of discrete elements in an energy domain—in [Chapters 02 graphs](#) and [03 ss](#), the electrical and mechanical energy domains. Also recall from [Chapter 04 emech](#) that energy transducing elements allow energy to flow among domains. In this chapter, we introduce fluid and thermal energy domains and discrete and transducing elements associated therewith.

2 The analogs between the mechanical and electrical systems from [Chapter 02 graphs](#) are expanded to include fluid and thermal systems. This generalization allows us to include, in addition to electromechanical systems, inter-domain systems including electrical, mechanical, fluid, and thermal systems.

3 This chapter begins by defining discrete lumped-parameter elements for fluid and thermal systems. We then categorize these into energy source, energy storage (A-type and T-type), and energy dissipative (D-type) elements, allowing us to immediately construct linear graphs and normal trees in the manner of [Chapter 02 graphs](#). Then we can directly apply the methods of [Chapter 03 ss](#) to construct state-space models of systems that include fluid and thermal elements.