08.2 thermoflu.therm Thermal system elements

1 Systems in which heat flow is of interest are called **thermal systems**. For instance, heat generated by an engine or a server farm flows through several bodies via the three modes of heat transfer: **conduction**, **convection**, and **radiation**. This is, of course, a dynamic process.

2 A detailed model would require a spatial continuum. However, we are often concerned with, say, the maximum temperature an engine will reach for different speeds or the maximum density of a server farm while avoiding overheating. Or, more precisely, *how* a given heat generation affects the temperature response of system components.

3 As with electrical, mechanical, and fluid systems, we can describe thermal systems as consisting of discrete lumped-parameter elements. The dynamic models that can be developed from considering these elements are often precisely the right granularity for system-level design.

4 We now introduce a few lumped-parameter elements for modeling thermal systems. Let a **heat flow rate** q (SI units W) and **temperature** T (SI units K or C) be input to a port in a thermal element. There are three structural differences between thermal systems and the other types we've considered. We are confronted with the **first**, here, when we consider that heat power is typically *not* considered to be the product of two variables; rather, the heat flow rate q is *already power*:

$$\mathcal{P}(t) = q(t). \tag{1}$$

A thermal element has two distinct locations between which its temperature drop is defined. We call a reference temperature **ground**.

5 The **heat** energy H of a system with initial heat $H(0)$ is

$$H(t) = \int_0^t \mathcal{P}(\tau) d\tau + H(0). \tag{2}$$

6 We now consider an element that can store energy, called an **energy storage element**; an element that resists power flow; and two elements that can supply power from outside a system, called **source elements**. The

second difference is that there is only one type of energy storage element in the thermal domain.

Thermal capacitors

7 When heat is stored in an object, it stores potential energy via its temperature T. This is analogous to how an electronic capacitor stores its energy via its voltage. For this reason, we call such thermal elements **thermal capacitors**.

8 A linear thermal capacitor with thermal capacitance C (SI units J/K), temperature T, and heat H has the constitutive equation

$$H = CT. \tag{3}$$

Once again, time-differentiating the constitutive equation gives us the elemental equation:

9 The thermal capacitance C is an **extensive property**—that is, it depends on the amount of its substance. This is opposed to the **specific heat capacity** c (units J/K/kg), an **intensive property**: one that is independent of the amount of its substance. These quantities are related for an object of mass m by the equation

$$C = mc. \tag{4}$$

Thermal resistors

10 **Thermal resistors** are defined as elements for which the heat flowrate q through the element is a monotonic function of the temperature drop T across it. **Linear thermal resistors** have constitutive equation (and, it turns out, elemental equation)

$$q = \frac{1}{R}T \tag{5}$$

where R is called the **thermal resistance**.

11 Thermal resistors do *not* dissipate energy from the system, which is the **third** difference between thermal and other energy domains we've considered. After all, the other "resistive" elements all dissipate energy *to heat*. Rather than dissipate energy, they simply impede its flow.

12 All three modes of heat transfer are modeled by thermal resistors, but only two of them are well-approximated as linear for a significant range of temperature.

conduction Heat conduction is the transfer of heat through an object's microscopic particle interaction.[1] It is characterized by a thermal resistance

$$R = \frac{L}{\rho A}, \tag{6}$$

where L is the length of the object *in the direction of heat transfer*, A is the transverse cross-sectional area, and ρ is the material's **thermal conductivity** (SI units W/K/m).[2]

convection Heat convection is the transfer of heat via **fluid advection**: the bulk motion of a fluid. It is characterized by a thermal resistance

$$R = \frac{1}{hA}, \tag{7}$$

where h is the **convection heat transfer coefficient** (SI units W/m^2/K) and A is the area of fluid-object contact (SI units m^2). The convection heat transfer coefficient h is highly and nonlinearly dependent on the velocity of the fluid. Furthermore, the geometry of the objects and the fluid composition affect h.

radiation Radiative heat transfer is electromagnetic radiation emitted from one body and absorbed by another. For T_1 the absolute temperature of a "hot" body, T_2 the absolute temperature of a "cold"

[1] We use the term "object" loosely, here, to mean a grouping of continuous matter in any phase.

[2] Thermal resistance can also be defined as an intensive property ρ^{-1}, the reciprocal of the thermal conductivity. Due to our lumped-parameter perspective, we choose the extensive definition.

body, ε the **effective emissivity/absorptivity**,[3] and A the area of the exposed surfaces, the heat transfer is characterized by

$$q = \varepsilon\sigma A(T_1^4 - T_2^4), \tag{8}$$

where σ is the **Stefan-Boltzmann constant**

$$\sigma = 5.67 \cdot 10^{-8}\ \frac{W}{m^2K^4}. \tag{9}$$

Clearly, this heat transfer is highly nonlinear. Linearization of this heat transfer is problematic because the temperature difference T between the bodies does not appear in the expression. For many system models, radiative heat transfer is assumed negligible. We must be cautious with this assumption, however, especially when high operating temperatures are anticipated.

Heat flow rate and temperature sources

13 Thermal sources include many physical processes—almost everything generates heat!

14 An **ideal heat flow rate source** is an element that provides arbitrary heat flow rate Q_s to a system, independent of the temperature across it, which depends on the system.

15 An **ideal temperature source** is an element that provides arbitrary temperature T_s to a system, independent of the heat flow rate through it, which depends on the system.

Generalized element and variable types

16 In keeping with the definitions of Chapter 01 intro, temperature T is an **across-variable** and heat flow rate q is a **through-variable**.

17 Consequently, the thermal capacitor is considered an **A-type** energy storage element. A thermal resistor is considered to be a **D-type** energy dissipative element, although it does not actually dissipate energy. It does,

[3] The parameter ε is taken to be the combined "gray body" emissivity/absorptivity. Consult a heat transfer text for details.

however, *resist* its flow and relates its across- and through-variables *algebraically*, both signature characteristics of D-type elements.

18 Temperature sources are, then, across-variable sources and heat flow rate sources are through-variable sources.

Example 08.2 thermoflu.therm-1

re: thermal system graph

Careless Carlton left a large pot of water boiling on the stove. Worse, a cast-iron pan is bumped so that it is in solid contact with the pot *and* his glass fish tank, which was carelessly left next to the stove, as shown in Fig. therm.1. Draw a linear graph of the sad situation to determine what considerations determine if Careless Carlton's fish goes from winner to dinner.

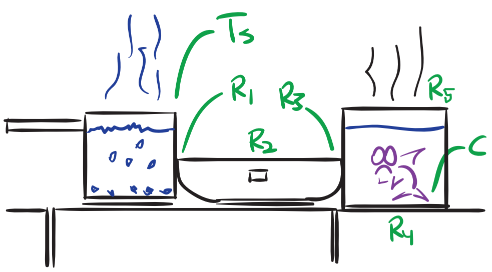

Figure therm.1: Careless Carlton's fish's sad situation.