10.4 freq.bodesimp Bode plots for simple transfer functions

1 This lecture also appears in *Control: an introduction*.

2 It turns out that bode plots, both magnitude and phase, given their logarithmic scale (recall that the ω-axes are also plotted logarithmically), are quite asymptotic to straight-lines for first- and second-order systems. Furthermore, higher-order system transfer functions can be re-written as the *product* of those of first-and second-order. For instance,

$$H(s) = \frac{\_\, s + \_}{s^3 + \_\, s^2 + \_\, s + \_} \tag{1a}$$

$$= \_ \cdot (\_\, s + 1) \cdot \frac{1}{\_\, s + 1} \cdot \frac{1}{s^2 + \_\, s + \_} \tag{1b}$$

3 Recall (from, for instance, phasor representation) that for products of complex numbers, *phases* ϕ_i *add* and *magnitudes* M_i *multiply*. For instance,

$$M_1 \angle \phi_1 \cdot \frac{1}{M_2 \angle \phi_2} \cdot \frac{1}{M_3 \angle \phi_3} = \frac{M_1}{M_2 M_3} \angle (\phi_1 - \phi_2 - \phi_3). \tag{2}$$

And if one takes the logarithm of the magnitudes, they add; for instance,

$$\log \frac{M_1}{M_2 M_3} = \log M_1 - \log M_2 - \log M_3. \tag{3}$$

There is only one more link in the chain: first- and second-order Bode plots depend on a handful of parameters that can be found *directly from transfer functions*. There is no need to compute $|H(j\omega_0)|$ and $\angle H(j\omega_0)$!

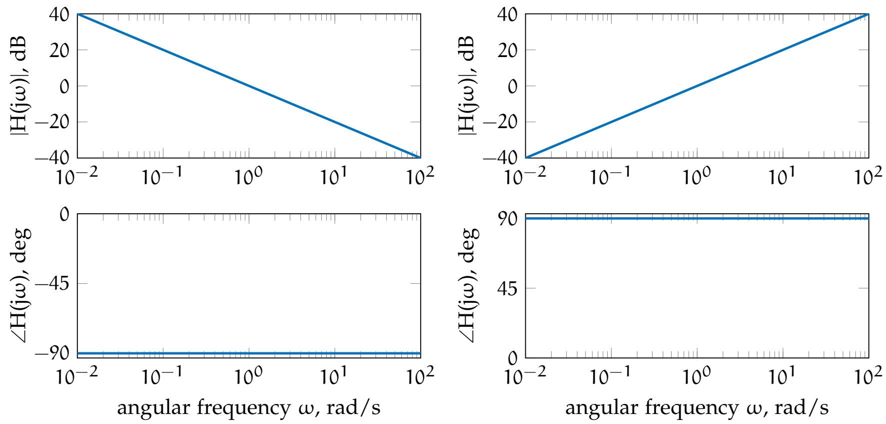
4 In a manner similar to Example 10.3 freq.bode-1, we construct Bode plots for several simple transfer functions in this lecture. Once we have these simple "building blocks," we will be able to construct sketches of higher-order systems by graphical addition because logarithmic magnitudes and phases combine by summation, as shown in Lec. 10.5 freq.bodesketch.

Constant gain

5 For a transfer function that is simply a constant real gain $H(s) = K$, the frequency response function is trivially $H(j\omega) = K$. Its magnitude $|H(j\omega)| = |K|$. For positive gain K, the phase is $\angle H(j\omega) = 0$, and for negative K, the phase is $\angle H(j\omega) = 180\,\text{deg}$.

Pole and zero at the origin

6 In Example 10.3 freq.bode-1, we have already demonstrated how to derive from the transfer function $H(s) = s$, a zero at the origin, the frequency response function plotted in Fig. bodesimp.1. Similarly, for $H(s) = 1/s$, a pole at the origin, the frequency response function plotted in Fig. bodesimp.1.



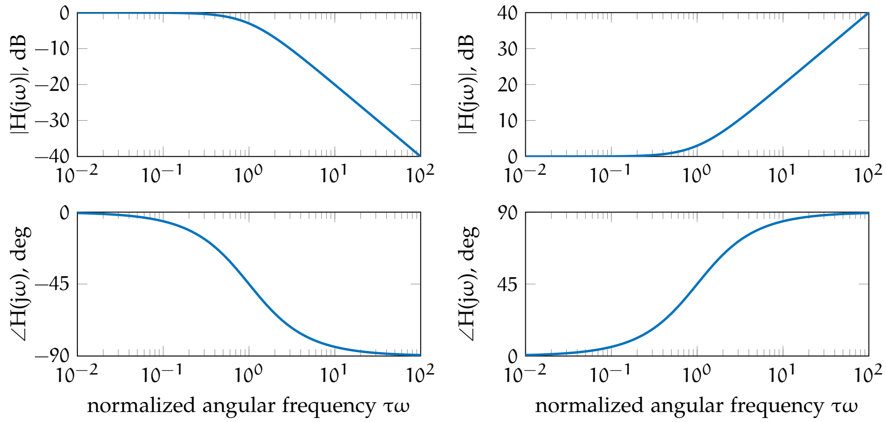
(a) $H(s) = 1/s$. (b) $H(s) = s$.

Figure bodesimp.1: Bode plots for (a) a pole at the origin and (b) a zero at the origin.

Real pole and real zero

7 The derivations for real poles and zeros are not included, but the resulting Bode plots are shown in Fig. bodesimp.2.

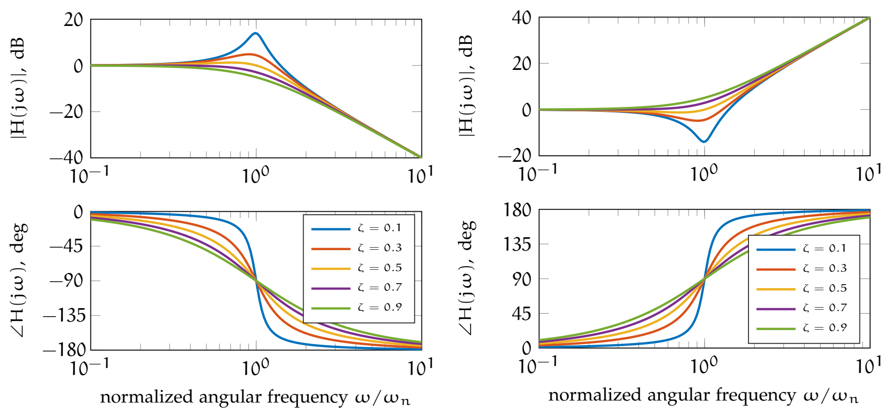
Complex conjugate pole pairs and zero pairs



(a) $H(s) = 1/(\tau s + 1)$. (b) $H(s) = \tau s + 1$.

Figure bodesimp.2: Bode plots for (a) a single real pole and (b) a single real zero.

8 The derivations for complex conjugate pole pairs and zero pairs are not included, but the resulting Bode plots are shown in Fig. bodesimp.3.



(a) $H(s) = \omega_n^2/(s^2 + 2\zeta\omega_n s + \omega_n^2)$. (b) $H(s) = \left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)/\omega_n^2$.

Figure bodesimp.3: Bode plots for (a) a complex conjugate pole pair and (b) a complex conjugate zero pair.

9 This lecture also appears in *Control: an introduction*.