## 10.7 freq.exe Exercises for Chapter 10 freq

## Exercise 10.1 gauche

Consider a system with i/o ODE

$$
\ddot{y}+a \dot{y}+b y=b u
$$

for constants $\mathrm{a}, \mathrm{b} \in \mathbb{R}$.

1. Derive the frequency response function $\mathrm{H}(\mathrm{j} \omega)$ and the transfer function $\mathrm{H}(\mathrm{s})$. Hint: either can be found from the other.
2. Let $\mathfrak{u}(\mathrm{t})=7 \cos (5 \mathrm{t}+3)$. What is the steady state forced response $y(t)$ in terms of $\mathrm{a}, \mathrm{b}$ ? Hint: this shouldn't require much computation.
3. Now let $u(t)=3 \delta(t)$, an impulse. What is the impulse response $y(t)$ in terms of the inverse Fourier transform $\mathcal{F}^{-1}$ and $\mathrm{H}(\mathrm{j} \omega)$ ? Do not substitute in for $\mathrm{H}(\mathrm{j} \omega)$ or inverse transform.
4. Use computer software to plot the Bode plot of $H(j \omega)$ for $a=b=1$.
5. For $b=1$, for what range of $a$ will there be a complex conjugate pair of poles? ${ }^{3}$ Hint: consider comparing the transfer function derived in part (a) to the standard form of the second-order transfer function in Fig. bodesimp.3a.

## Exercise 10.2 tickle

Let a transfer function H be

$$
\frac{10(s+100)}{s^{2}+2 s+100} .
$$

Use H to respond to the following questions and imperatives.

[^0]a. Write H as a product of standard-form transfer functions.
b. Find the frequency response function $\mathrm{H}(\mathrm{j} \omega)$ without simplifying.
c. Use the axes below to sketch the Bode plot of H .


## Exercise 10.3 me

Let a transfer function H be

$$
H(s)=\frac{1000(s+10)}{(s+100)(s+1000)} .
$$

Use H to respond to the following questions and imperatives.
a. Write H as a product of standard-form transfer functions.
b. Find the frequency response function $\mathrm{H}(\mathrm{j} \omega)$ without simplifying.
c. Use the axes below to sketch the Bode plot of H.


## Exercise 10.4 elmo

Consider a system with transfer function

$$
H(s)=\frac{100(s+9)}{(s+5)(s+6)\left(s^{2}+8 s+32\right)} .
$$

a. Identify the poles and zeros of H .
b. Derive the frequency response function $\mathrm{H}(\mathrm{j} \omega)$. Do not simplify the expression.
c. Create a Bode plot of H.
d. Let the system have sinusoidal input $u(t)=2 \cos (3 t)$. What is the steady-state system output $y(t)$ ?
e. Let the system have the same sinusoidal input as previously. Simulate its forced response for nine seconds and plot it.

Part $V$

## Laplace analysis

11 lap


[^0]:    ${ }^{3}$ The following statements are equivalent. A second-order system

    - has a complex conjugate pair of poles,
    - has a complex conjugate pair of the characteristic equation,
    - has a complex conjugate pair of eigenvalues, and
    - is underdamped.

