# 10.7 freq.exe Exercises for Chapter 10 freq

### Exercise 10.1 gauche

Consider a system with i/o ODE

$$\ddot{y} + a\dot{y} + by = bu \tag{1}$$

for constants  $a, b \in \mathbb{R}$ .

- 1. Derive the frequency response function  $H(j\omega)$  and the transfer function H(s). *Hint: either can be found from the other*.
- 2. Let  $u(t) = 7\cos(5t + 3)$ . What is the steady state forced response y(t) in terms of a, b? *Hint: this shouldn't require much computation*.
- 3. Now let  $u(t) = 3 \delta(t)$ , an impulse. What is the impulse response y(t) in terms of the inverse Fourier transform  $\mathcal{F}^{-1}$  and  $H(j\omega)$ ? Do *not* substitute in for  $H(j\omega)$  or inverse transform.
- 4. Use computer software to plot the Bode plot of  $H(j\omega)$  for a=b=1.
- 5. For b = 1, for what range of a will there be a complex conjugate pair of poles?<sup>3</sup> Hint: consider comparing the transfer function derived in part (a) to the standard form of the second-order transfer function in Fig. bodesimp.3a.

#### Exercise 10.2 tickle

Let a transfer function H be

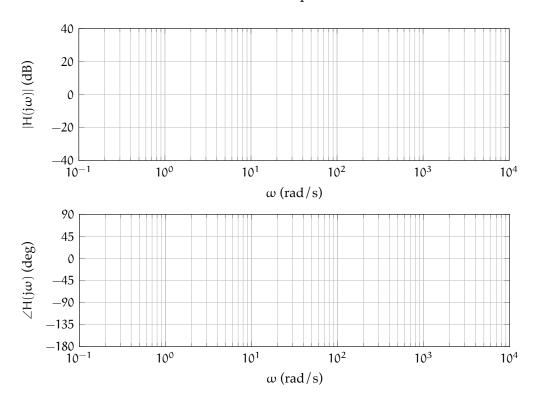
$$\frac{10(s+100)}{s^2+2s+100}.$$
 (2)

Use H to respond to the following questions and imperatives.

- has a complex conjugate pair of poles,
- has a complex conjugate pair of the characteristic equation,
- has a complex conjugate pair of eigenvalues, and
- is underdamped.

<sup>&</sup>lt;sup>3</sup>The following statements are equivalent. A second-order system

- a. Write H as a product of standard-form transfer functions.
- b. Find the frequency response function  $H(j\omega)$  without simplifying.
- c. Use the axes below to sketch the Bode plot of H.



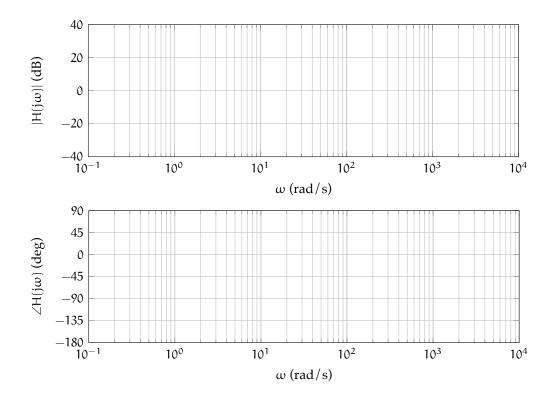
#### Exercise 10.3 me

Let a transfer function H be

$$H(s) = \frac{1000(s+10)}{(s+100)(s+1000)}.$$

Use H to respond to the following questions and imperatives.

- a. Write H as a product of standard-form transfer functions.
- b. Find the frequency response function  $H(j\omega)$  without simplifying.
- c. Use the axes below to sketch the Bode plot of H.



#### Exercise 10.4 elmo

Consider a system with transfer function

$$H(s) = \frac{100(s+9)}{(s+5)(s+6)(s^2+8s+32)}.$$

- a. Identify the poles and zeros of H.
- b. Derive the frequency response function  $H(j\omega)$ . Do *not* simplify the expression.
- c. Create a Bode plot of H.
- d. Let the system have sinusoidal input  $u(t)=2\cos(3t)$ . What is the steady-state system output y(t)?
- e. Let the system have the same sinusoidal input as previously. Simulate its forced response for nine seconds and plot it.

Part V

Laplace analysis

## 11 lap