10.7 freq.exe Exercises for Chapter 10 freq

Exercise 10.1 gauche

Consider a system with i/o ODE

$$\ddot{y} + a\,\dot{y} + b\,y = b\,u$$

for constants $a, b \in \mathbb{R}$.

- Derive the frequency response function H(jω) and the transfer function H(s). *Hint: either can be found from the other.*
- 2. Let $u(t) = 7\cos(5t + 3)$. What is the steady state forced response y(t) in terms of a, b? *Hint: this shouldn't require much computation*.
- 3. Now let $u(t) = 3 \delta(t)$, an impulse. What is the impulse response y(t) in terms of the inverse Fourier transform \mathcal{F}^{-1} and $H(j\omega)$? Do *not* substitute in for $H(j\omega)$ or inverse transform.
- 4. Use computer software to plot the Bode plot of $H(j\omega)$ for a = b = 1.
- 5. For b = 1, for what range of a will there be a complex conjugate pair of poles?³ *Hint: consider comparing the transfer function derived in part (a) to the standard form of the second-order transfer function in Fig. bodesimp.3a.*

Exercise 10.2 tickle

Let a transfer function H be

$$\frac{10(s+100)}{s^2+2\,s+100}.$$
 (2)

Use H to respond to the following questions and imperatives.

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³The following statements are equivalent. A second-order system

[•] has a complex conjugate pair of poles,

has a complex conjugate pair of the characteristic equation,

has a complex conjugate pair of eigenvalues, and

[•] is underdamped.

- a. Write H as a product of standard-form transfer functions.
- b. Find the frequency response function $H(j\omega)$ *without* simplifying.
- c. Use the axes below to sketch the Bode plot of H.



Exercise 10.3 me

Let a transfer function H be

$$H(s) = \frac{1000(s+10)}{(s+100)(s+1000)}.$$

Use H to respond to the following questions and imperatives.

- a. Write H as a product of standard-form transfer functions.
- b. Find the frequency response function $H(j\omega)$ *without* simplifying.
- c. Use the axes below to sketch the Bode plot of H.



Exercise 10.4 elmo

Consider a system with transfer function

$$H(s) = \frac{100(s+9)}{(s+5)(s+6)(s^2+8s+32)}.$$

- a. Identify the poles and zeros of H.
- b. Derive the frequency response function $H(j\omega)$. Do *not* simplify the expression.
- c. Create a Bode plot of H.
- d. Let the system have sinusoidal input $u(t) = 2\cos(3t)$. What is the steady-state system output y(t)?
- e. Let the system have the same sinusoidal input as previously. Simulate its forced response for nine seconds and plot it.

Exercise 10.5 hum

In many measurement systems, an **interference signal** (i.e., an uncontrolled, undesirable signal that appears in a signal) that often appears in measurements is the **mains hum**, which arises from the mains power grid. Its fundamental frequency is $f_1 = 60$ Hz, and much smaller-amplitude components appear at higher harmonics.

Consider the following measurement system. A pressure sensor has transfer function

$$G(s) = \frac{1 \cdot 10^3}{s^2 + 1 \cdot 10^3 s + 1 \cdot 10^6}.$$

with units V/Pa. The desired measurement signal has maximum angular frequency $\omega_{sig} = 100 \text{ rad/s}$. Mains hum is observed in the measurement signal with magnitude m = 0.5 V.

Design a first-order low-pass filter

$$H(s) = \frac{1}{s/\omega_b + 1}$$
(3)

for the output of the sensor with the following steps:

- a. Derive the filter frequency response function $H(j\omega)$.
- b. Compute the magnitude $|H(j\omega)|$ of the frequency response function.
- c. Solve for the break frequency ω_b such that $|H(j\omega_{sig})| = 0.97$. This design will leave the desired signal only lightly attenuated but will further attenuate the mains hum.
- d. Compute the attenuation of the mains hum amplitude $|H(j60 \cdot 2\pi)|$ and the corresponding filter output $|H(j60 \cdot 2\pi)|m$.
- e. Compute the steady-state filtered output amplitude for a sensor input $1\sin(\omega_{sig}t)$ kPa.
- f. Suppose a greater attenuation of the mains hum interference is desired. How could the filter H(s) be altered to reduce it further *without* significantly attenuating the desired signal?
- g. Find the sensor's natural frequency ω_n and damping ratio ζ . Its peak output magnitude occurs at a frequency of $\omega_p = \omega_n \sqrt{1 2\zeta^2}$. Compute ω_p for the sensor G.

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EXERCISES FOR CHAPTER 10 FREQ

h. Compute the steady-state filtered output amplitude for a sensor input $1\sin(\omega_p t)$ kPa. Explain why it is greater than the filtered output magnitude at ω_{sig} from part e.

Part V

Laplace analysis

11 lap