13.1 imp.ip Input impedance and admittance

1 We now introduce a generalization of the familiar impedance and admittance of electrical circuit analysis, in which system behavior can be expressed algebraically instead of differentially. We begin with generalized input impedance.

2 Consider a system with a source, as shown in Fig. ip.1. The source can be either an across- or a through-variable source. The ideal source specifies either $\mathcal{V}_{\text{in}}$ or $\mathcal{F}_{\text{in}}$, and the other variable depends on the system.

3 Let a source variables have Laplace transforms $\mathcal{V}_{\text{in}}(s)$ and $\mathcal{F}_{\text{in}}(s)$. We define the system's **input impedance** Z and **input admittance** Y to be the Laplace-domain ratios

$$Z(s) = \frac{\mathcal{V}_{\text{in}}(s)}{\mathcal{F}_{\text{in}}(s)} \quad \text{and} \quad Y(s) = \frac{\mathcal{F}_{\text{in}}(s)}{\mathcal{V}_{\text{in}}(s)}. \tag{1}$$

Clearly,

Both Z and Y can be considered **transfer functions**: for a through-variable source $\mathcal{F}_{\text{in}}$, the impedance Z is the transfer function to across-variable $\mathcal{V}_{\text{in}}$; for an across-variable source $\mathcal{V}_{\text{in}}$, the admittance Y is the transfer function to through-variable $\mathcal{F}_{\text{in}}$. Often, however, we use the more common impedance Z to characterize systems with either type of source.

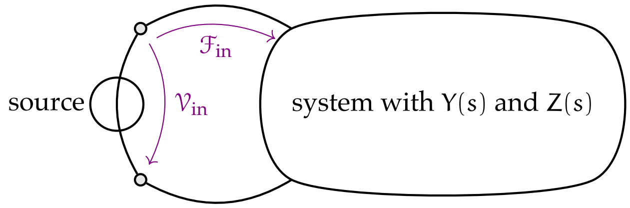


Figure ip.1:

4 Note that Z and Y are **system properties**, not properties of the source. An impedance or admittance can characterize a system of interconnected elements, or a system of a single element, as the next section explores.

Impedance of ideal passive elements

5 The impedance and admittance of a single, ideal, one-port element is defined from the Laplace transform of its elemental equation.

Generalized capacitors A **generalized capacitor** has elemental equation

$$\frac{d\mathcal{V}_C(t)}{dt} = \frac{1}{C}\mathcal{F}_C(t), \tag{2}$$

the Laplace transform of which is

$$s\mathcal{V}_C(s) = \frac{1}{C}\mathcal{F}_C(s), \tag{3}$$

which can be solved for impedance $Z_C = \mathcal{V}_C/\mathcal{F}_C$ and admittance $Y_C = \mathcal{F}_C/\mathcal{V}_C$:

Generalized inductors A **generalized inductor** has elemental equation

$$\frac{d\mathcal{F}_L(t)}{dt} = \frac{1}{L}\mathcal{F}_L(t), \tag{4}$$

the Laplace transform of which is

$$s\mathcal{F}_L(s) = \frac{1}{L}\mathcal{V}_L(s), \tag{5}$$

which can be solved for impedance $Z_L = \mathcal{V}_L/\mathcal{F}_L$ and admittance $Y_L = \mathcal{F}_L/\mathcal{V}_L$:

Generalized resistors A **generalized resistor** has elemental equation

$$\mathcal{V}_R(t) = \mathcal{F}_R(t)R, \tag{6}$$

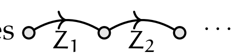
the Laplace transform of which is

$$\mathcal{V}_R(s) = \mathcal{F}_R(s)R, \tag{7}$$

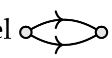
which can be solved for impedance $Z_R = \mathcal{V}_R/\mathcal{F}_R$ and admittance $Y_R = \mathcal{F}_R/\mathcal{V}_R$:

6 For a summary of the impedance of one-port elements, see Table els.1.

Impedance of interconnected elements

7 As with electrical circuits, impedances of linear graphs of interconnected elements can be combined in two primary ways: in parallel or in series.
8 Elements sharing the same through-variable are said to be in **series** connection. N elements connected in series  have equivalent impedance Z and admittance Y:

$$Z(s) = \sum_{i=1}^{N} Z_i(s) \quad \text{and} \quad Y(s) = 1 \Big/ \sum_{i=1}^{N} 1/Y_i(s) \tag{8}$$

9 Conversely, elements sharing the same across-variable are said to be in **parallel** connection. N elements connected in parallel  have equivalent impedance Z and admittance Y:

$$Z(s) = 1 \Big/ \sum_{i=1}^{N} 1/Z_i(s) \quad \text{and} \quad Y(s) = \sum_{i=1}^{N} Y_i(s). \tag{9}$$

Example 13.1 imp.ip-1

re: input impedance of a simple circuit

For the circuit shown, find the input impedance.