## 13.2 imp.2port Impedance with two-port elements

1 The two types of energy transducing elements, **transformers** and **gyrators**, "reflect" or "transmit" impedance through themselves, such that they are "felt" on the other side.

2 For a transformer, the elemental equations are

$$\mathcal{V}_2(t) = \mathcal{V}_1(t)/\mathsf{TF} \quad \text{and} \quad \mathcal{F}_2(t) = -\mathsf{TF}\mathcal{F}_1(t),$$
(1)

the Laplace transforms of which are

$$\mathcal{V}_2(s) = \mathcal{V}_1(s)/\mathsf{TF}$$
 and  $\mathcal{F}_2(s) = -\mathsf{TF}\mathcal{F}_1(s)$ . (2)



Figure 2port.1:

3 If, on the 2-side, the input impedance is  $Z_3$ , as in Fig. 2port.1, the equations of Eq. 2 are subject to the continuity and compatibility equations

$$\mathcal{V}_2 = \mathcal{V}_3 \quad \text{and} \quad \mathcal{F}_2 = -\mathcal{F}_3.$$
 (3)

Substituting these into Eq. 2 and solving for  $V_1$  and  $\mathcal{F}_1$ ,

$$\mathcal{V}_1 = \mathsf{TFV}_3 \quad \text{and} \quad \mathcal{F}_1 = \mathcal{F}_3 / \mathsf{TF}.$$
 (4)

The elemental equation for element 3 is  $V_3 = \mathcal{F}_3 Z_3$ , which can be substituted into the through-variable equation to yield

4 Working our way back from  $V_3$  to  $V_1$ , we apply the compatibility equation  $V_2 = V_3$  and the elemental equation  $V_2 = V_1/\text{TF}$ , as follows:

Solving for the **effective input impedance** Z<sub>1</sub>,

$$\mathsf{Z}_1 \equiv \frac{\mathcal{V}_1(\mathsf{s})}{\mathcal{F}_1(\mathsf{s})} \tag{5}$$

$$= TF^2Z_3.$$
 (6)



Figure 2port.2:

**5** For a **gyrator** with gyrator modulus GY, in the configuration shown in Fig. 2port.2, a similar derivation yields the **effective input impedance** Z<sub>1</sub>,

$$\mathsf{Z}_1 = \mathsf{G}\mathsf{Y}^2/\mathsf{Z}_3. \tag{7}$$

## Example 13.2 imp.2port-1

Draw a linear graph of the fluid system. What is the input impedance for an input force to the piston?

re: input impedance of fluid system with transducer

