## 13.2 imp.2port Impedance with two-port elements

**1** The two types of energy transducing elements, **transformers** and **gyrators**, "reflect" or "transmit" impedance through themselves, such that they are "felt" on the other side.

**2** For a **transformer**, the elemental equations are

$$\mathcal{V}_2(t) = \mathcal{V}_1(t)/\mathrm{TF} \quad \text{and} \quad \mathcal{F}_2(t) = -\mathrm{TF}\mathcal{F}_1(t), \tag{1}$$

the Laplace transforms of which are

$$\mathcal{V}_2(s) = \mathcal{V}_1(s)/\mathrm{TF} \quad \text{and} \quad \mathcal{F}_2(s) = -\mathrm{TF}\mathcal{F}_1(s). \tag{2}$$

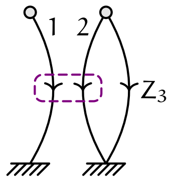



**Figure 2port.1:**

**3** If, on the 2-side, the input impedance is $Z_3$, as in Fig. 2port.1, the equations of Eq. 2 are subject to the continuity and compatibility equations

$$\mathcal{V}_2 = \mathcal{V}_3 \quad \text{and} \quad \mathcal{F}_2 = -\mathcal{F}_3. \tag{3}$$

Substituting these into Eq. 2 and solving for $\mathcal{V}_1$ and $\mathcal{F}_1$,

$$\mathcal{V}_1 = \mathrm{TF}\mathcal{V}_3 \quad \text{and} \quad \mathcal{F}_1 = \mathcal{F}_3/\mathrm{TF}. \tag{4}$$

The elemental equation for element 3 is $\mathcal{V}_3 = \mathcal{F}_3 Z_3$, which can be substituted into the through-variable equation to yield

**4** Working our way back from $\mathcal{V}_3$ to $\mathcal{V}_1$, we apply the compatibility equation $\mathcal{V}_2 = \mathcal{V}_3$ and the elemental equation $\mathcal{V}_2 = \mathcal{V}_1/\mathrm{TF}$, as follows:

Solving for the **effective input impedance** $Z_1$,

$$Z_1 \equiv \frac{\mathcal{V}_1(s)}{\mathcal{F}_1(s)} \tag{5}$$

$$= TF^2 Z_3. \tag{6}$$

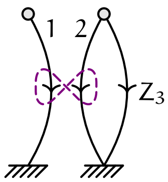



**Figure 2port.2:**

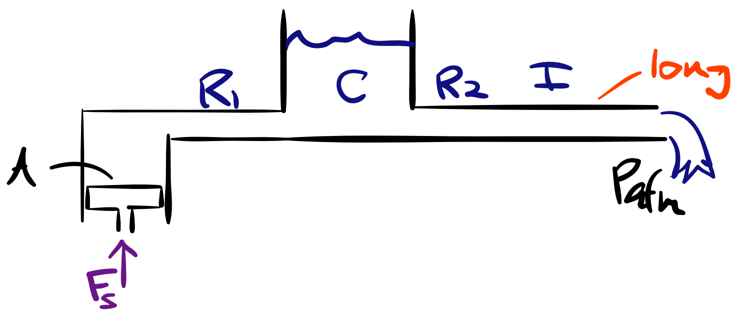
5 For a **gyrator** with gyrator modulus GY, in the configuration shown in Fig. 2port.2, a similar derivation yields the **effective input impedance** $Z_1$,

$$Z_1 = GY^2/Z_3. \tag{7}$$

## Example 13.2 imp.2port-1

**re: input impedance of fluid system with transducer**

Draw a linear graph of the fluid system. What is the input impedance for an input force to the piston?


R1
C
R2
I
long
A
Fs
Patm