13.3 imp.tf Transfer functions via impedance

1 Now the true power of impedance-based modeling is revealed: we can skip a time-domain model (e.g. state-space or io differential equation) and derive a transfer-function model, directly! Before we do, however, let's be sure to recall that a transfer-function model concerns itself with the **forced response** of a system, ignoring the free response. If we care to consider the free response, we can convert the transfer function model to an io differential equation and solve it.

2 There are two primary ways impedance-based modeling is used to derive transfer functions. The first and most general is described, here. The second is a shortcut most useful for relatively simple systems; it is described in Lec. 13.6 imp.divide.

3 In what follows, it is important to recognize that, in the Laplace-domain, every elemental equation is just[1]

$$\mathcal{V} = \mathcal{F}Z, \tag{1}$$

where the across-variable, through-variable, and impedance are all element-specific.

4 This algorithm is very similar to that for state-space models from linear graph models, presented in Lec. 03.4 ss.nt2ss. In the following, we consider a connected graph with B branches, of which S are sources (split between through-variable sources S_T and across S_A). There are $2B - S$ unknown across- and through-variables, so that's how many equations we need. We have $B - S$ elemental equations and for the rest we will write continuity and compatibility equations. N is the number of nodes.

1. Derive $2B - S$ independent Laplace-domain, algebraic equations from Laplace-domain elemental, continuity, and compatibility equations.
    a) Draw a **normal tree**.

[1] In electronics, this is sometimes called "generalized Ohm's law."

b) Write a Laplace-domain **elemental equation** for each passive element.[2]

c) Write a **continuity equation** for each passive branch by drawing a contour intersecting that and no other branch.[3]

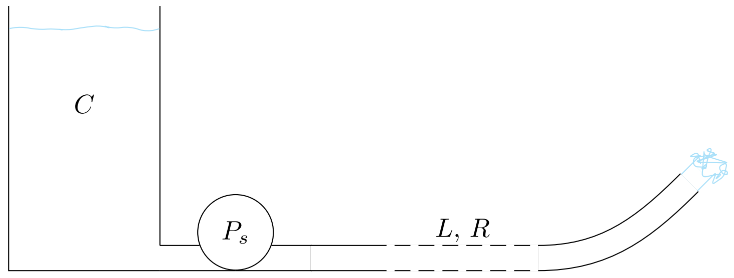
d) Write a **compatibility equation** for each passive link by temporarily "including" it in the tree and finding the compatibility equation for the resulting loop.[4]

2. Solve the *algebraic* system of $2B$ equations and $2B$ unknowns for outputs in terms of inputs, only. Sometimes, solving for *all* unknowns via the usual methods is easier than trying to cherry-pick the desired outputs.
3. The solution for each output Y_i depends on zero or more inputs U_j. To solve for the transfer function Y_i/U_j, set $U_k = 0$ for all $k \neq j$, then divide both sides of the equation by U_j.

Example 13.3 imp.tf-1

re: firehose

For the schematic of a fire hose connected to a fire truck's reservoir C via pump input P_s, use impedance methods to find the transfer function from P_s to the velocity of the spray. Assume the nozzle's cross-sectional area is A.



[2] There will be $B - S$ elemental equations.
[3] There will be $N - 1 - S_A$ independent continuity equations.
[4] There will be $B - N + 1 - S_T$ independent compatibility equations.