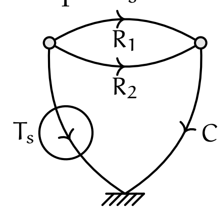
13.7 imp.exe Exercises for Chapter 13 imp

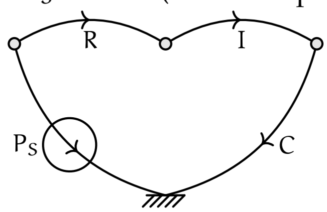
Exercise 13.1 tile

Use the linear graph below of a thermal system to **(a)** derive the transfer function $T_{R_2}(s)/T_s(s)$, where T_s is the input temperature and T_{R_2} is the temperature across the thermal resistor R_2. *Use impedance methods*. And **(b)** derive the input impedance the input T_s drives.



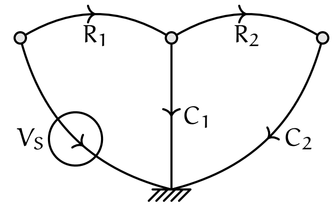
Exercise 13.2 granite

Use the linear graph below of a fluid system to **(a)** derive the transfer function $P_C(s)/P_S(s)$, where P_S is the input pressure and P_C is the pressure across the fluid capacitance C. *Use impedance methods and a divider rule is highly recommended*. (Simplify the transfer function.) And **(b)** derive the input impedance the input P_S drives. (Don't simplify the expression.)



Exercise 13.3 granted

Use the linear graph below of an electronic system to derive the transfer function $I_{R_1}(s)/V_S(s)$, where V_S is the input voltage and I_{R_1} is the current through the resistor R_1. (Simplify the transfer function.) Use an impedance method. *Hint: a divider method is recommended; without it, use of a computer is recommended.*



Exercise 13.4 concrete

Use the linear graph of a fluid system in Fig. exe.1 to derive the transfer function $Q_C(s)/P_S(s)$, where P_S is the input pressure and Q_C is the flowrate through the fluid capacitance C. *Use impedance methods*; a divider rule is recommended but not required. Identify all impedances but do *not* substitute them into the transfer function. ______/ 25 p.

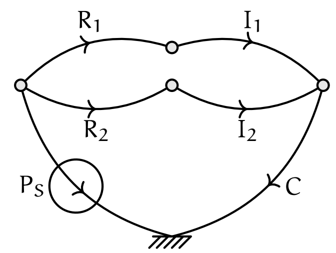


Figure exe.1: a fluid system linear graph.

Exercise 13.5 tableau

Consider an accelerometer that has transfer function

$$G(s) \equiv \frac{V_i(s)}{A(s)} = \frac{K_G\,\omega_{n_G}^2}{s^2 + 2\zeta_G \omega_{n_G} s + \omega_{n_G}^2}, \qquad (1)$$

where

- A is the input acceleration in m/s^2,
- V_i is the output voltage in V,
- $K_G = 0.1$ V/(m/s^2) is the gain,
- $\omega_{n_G} = 3000$ rad/s is the natural frequency, and

- $\zeta_G = 0.2$ is the damping ratio.

Perform a frequency domain analysis as follows.

a. Generate a Bode plot of $G(s)$.
b. At DC ($\omega = 0$ rad/s), compute the *magnitude* and *phase* of the frequency response function of the accelerometer.

Suppose there is a sinusoidal systematic noise signal at the input, with amplitude $a_{\text{noise}} = 1\ \text{m/s}^2$ and frequency $\omega_{\text{noise}} = 2900$ rad/s.[7]

c. Assuming there is only noise input, at the noise frequency ω_{noise}, compute the *amplitude* and *phase* of the voltage V_i at the output of the accelerometer. Why is the amplitude higher than it would have been at DC (use your Bode plot from Item a. to justify your answer).

To mitigate the systematic noise, we add a filter with transfer function $H(s)$ to the output of the accelerometer, as shown in Fig. exe.2.

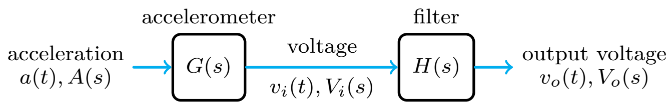


Figure exe.2: Accelerometer and filter block diagram.

By definition,

$$H(s) \equiv \frac{V_o(s)}{V_i(s)}. \tag{2}$$

Assume the filter and accelerometer *do not* dynamically load each other. The filter circuit diagram is shown in Fig. exe.3.

[7]Assume the input phase is zero.

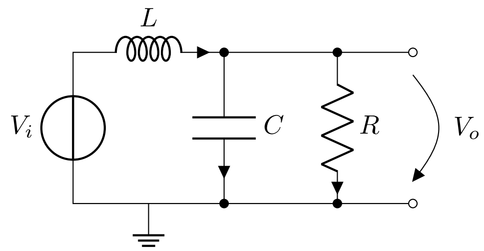


Figure exe.3: Filter circuit.

d. Draw a linear graph model of the filter circuit.
e. Use *impedance methods* to derive the transfer function $H(s)$ in terms of the circuit element parameters R, L, and C.
f. Find the filter's natural frequency ω_{n_H} and damping ratio ζ_H.[8]
g. Let $C = 0.001$ F. Design the filter by choosing R and L such that

$$\zeta_H = 1 \quad \text{and} \quad \omega_{n_H} = 1000 \text{ rad/s}. \tag{3}$$

h. Find the transfer function

$$\frac{V_o(s)}{A(s)} \tag{4}$$

with all parameters substituted. Simplify.
i. Generate a Bode plot for $V_o(s)/A(s)$.
j. Using the Bode plot of Item i., explain why we should expect the output from the systematic noise at ω_{noise} to be improved.
k. From the transfer function $V_o(s)/A(s)$, at the noise frequency ω_{noise}, compute the *amplitude* and *phase* of the output voltage V_o.
l. Compare the result from Item k. to the unfiltered voltage in Item c. by finding the ratio of the filtered amplitude over then unfiltered amplitude.
m. How could you augment the filter design to further reduce the systematic noise?

[8] Be cautious to make the denominator have the proper standard form $s^2 + 2\zeta_H \omega_{n_H} s + \omega_{n_H}^2$.

Exercise 13.6 gypsum

Respond to the following questions and imperatives with a sentence or two, equation, and/or a sketch.

a. Comment on the *stability* and *transient response characteristics* of a system with eigenvalues

$$-2, -5, -8 + j3, -8 - j3.$$

b. Consider an LTI system that, given input u_1, outputs y_1, and given input u_2, outputs y_2. If the input is $u_3 = 5u_1 - 6u_2$, what is the output y_3?
c. Consider a second-order system with natural frequency $\omega_n = 2$ rad/s and damping ratio $\zeta = 0.5$. What is the free response for initial condition $y(0) = 1$?
d. Two thermal elements with impedances Z_1 and Z_2 have a temperature source T_S applied across them *in series*. What is the transfer function from T_S to the heat Q_2 through Z_2?
e. Draw a linear graph of a pump (pressure source) flowing water through a long pipe into the bottom of a tank, which has a valve at its bottom from which the water flows.

Part VI

Nonlinear system analysis

14 nlin

1 Thus far, we've mostly considered *linear* system models. Many of the analytic tools we've developed—ODE solution techniques, superposition, eigendecomposition, stability analysis, impedance modeling, transfer functions, frequency response functions—do not apply to nonlinear systems. In fact, analytic solutions are unknown for most nonlinear system ODEs. And even basic questions are relatively hard to answer; for instance: is the system stable?

2 In this and the following chapters, we consider a few analytic and numerical techniques for dealing with nonlinear systems.

3 A state-space model has the general form

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t) \tag{1a}$$

$$\mathbf{y} = \underline{\hspace{6em}} \tag{1b}$$

where $\mathbf{f}$ and $\mathbf{g}$ are vector-valued functions that depend on the system. **Nonlinear state-space models** are those for which $\mathbf{f}$ is a ________________ functional of either $\mathbf{x}$ or $\mathbf{u}$. For instance, a state variable x_1 might appear as x_1^2 or two state variables might combine as $x_1 x_2$ or an input u_1 might enter the equations as $\log u_1$.

Autonomous and nonautonomous systems

4 An **autonomous system** is one for which $\mathbf{f}(\mathbf{x})$, with neither time nor input appearing explicitly. A **nonautonomous system** is one for which either t or $\mathbf{u}$ *do* appear explicitly in $\mathbf{f}$. It turns out that we can always write

nonautonomous systems as autonomous by substituting in $\mathbf{u}(t)$ and introducing an extra ______________ for t[1].

5 Therefore, without loss of generality, we will focus on ways of analyzing autonomous systems.

Equilibrium

6 An **equilibrium state** (also called a ________________) $\bar{\mathbf{x}}$ is one for which $d\mathbf{x}/dt = \mathbf{0}$. In most cases, this occurs only when the input $\mathbf{u}$ is a constant $\bar{\mathbf{u}}$ and, for time-varying systems, at a given time $\bar{t}$. For autonomous systems, equilibrium occurs when the following holds:

$$\_\_\_\_\_\_\_\_\_\_\_\_ \tag{2}$$

This is a system of nonlinear algebraic equations, which can be challenging to solve for $\bar{\mathbf{x}}$. However, frequently, several solutions—that is, equilibrium states—do exist.

[1] Strogatz **and** Dichter, 2016.