14.2 nlin.char Nonlinear system characteristics

1 Characterizing nonlinear systems can be challenging without the tools developed for ____________ system characterization. However, there are ways of characterizing nonlinear systems, and we'll here explore a few.

Those in-common with linear systems

2 As with linear systems, the **system order** is either the number of state-variables required to describe the system or, equivalently, the highest-order _______________ in a single scalar differential equation describing the system.

3 Similarly, nonlinear systems can have state variables that depend on ___________ alone or those that also depend on ____________ (or some other independent variable). The former lead to ordinary differential equations (ODEs) and the latter to partial differential equations (PDEs).

4 Equilibrium was already considered in Chapter 14 nlin.

Stability

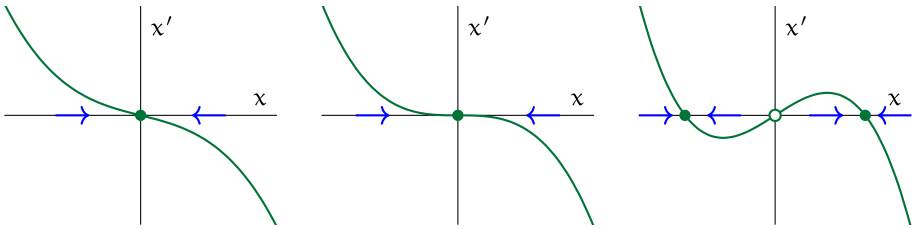
5 In terms of system performance, perhaps no other criterion is as important as ______________.

Definition 14 nlin.1: Stability

If x is perturbed from an equilibrium state $\overline{x}$, the response $x(t)$ can:

1. asymptotically return to $\overline{x}$ (asymptotically ____________),
2. diverge from $\overline{x}$ (______________), or
3. remain perturned or oscillate about $\overline{x}$ with a constant amplitude (________________ stable).

Notice that this definition is actually local: stability in the neighborhood of one equilibrium may not be the same as in the neighborhood of another.



(a) $r < 0$ (b) $r = 0$ (c) $r > 0$

Figure char.1: plots of x' versus x for Eq. 1.

6 Other than nonlinear systems' lack of linear systems' eigenvalues, poles, and roots of the characteristic equation from which to compute it, the primary difference between the stability of linear and nonlinear systems is that nonlinear system stability is often difficult to establish ____________. Using a linear system's eigenvalues, it is straightforward to establish stable, unstable, and marginally stable subspaces of state-space (via transforming to an eigenvector basis). For nonlinear systems, no such method exists. However, we are not without tools to explore nonlinear system stability. One mathematical tool to consider is ____________________, which is beyond the scope of this course, but has good treatments in[2] and[3].

Qualities of equilibria

7 Equilibria (i.e. stationary points) come in a variety of qualities. It is instructive to consider the first-order differential equation in state variable ________ with real constant ________:

$$x' = rx - x^3. \tag{1}$$

If we plot x' versus x for different values of r, we obtain the plots of Fig. char.1.

8 By definition, equilibria occur when $x' = 0$, so the x-axis crossings of Fig. char.1 are equilibria. The blue arrows on the x-axis show the ______________ of state change x', quantified by the plots. For both

[2] Brogan, 1991, Ch. 10.
[3] Choukchou-Braham **andothers**, 2013, App. A.

(a) and (b), only one equilibrium exists: $x = 0$. Note that the blue arrows in both plots point *toward* the equilibrium. In such cases—that is, when a ______________ exists around an equilibrium for which state changes point toward the equilibrium—the equilibrium is called an ______________ or ______________. Note that attractors are ______________.

9 Now consider (c) of Fig. char.1. When $r > 0$, three equilibria emerge. This change of the number of equilibria with the changing of a parameter is called a ______________. A plot of bifurcations versus the parameter is called a **bifurcation diagram**. The $x = 0$ equilibrium now has arrows that point ______________ from it. Such an equilibrium is called a ______________ or ______________ and is ______________. The other two equilibria here are (stable) attractors. Consider a very small initial condition $x(0) = \epsilon$. If $\epsilon > 0$, the repeller pushes away x and the positive attractor pulls x to itself. Conversely, if $\epsilon < 0$, the repeller again pushes away x and the negative attractor pulls x to itself.

10 Another type of equilibrium is called the ______________: one which acts as an attractor along some lines and as a repeller along others. We will see this type in the following example.

Example 14.2 nlin.char-1

re: Saddle bifurcation

Consider the dynamical equation

$$x' = x^2 + r \tag{2}$$

with r a real constant. Sketch x' vs x for negative, zero, and positive r. Identify and classify each of the equilibria.