14.3 nlin.exe Exercises for Chapter 14 nlin

Exercise 14.1 sigmund

Consider a nonlinear capacitor with constitutive equation relating charge q_C and voltage v_C:

$$q_C = k v_C^{3/2} \tag{1}$$

with k a positive constant.

a. Derive an elemental equation relating dv_C/dt and i_C for the nonlinear capacitor.
b. From the elemental equation, what is the voltage-dependent capacitance $C(v_C)$?
c. Consider the RLC-circuit of Fig. exe.1, which includes the nonlinear capacitor. Derive a nonlinear state-space equation with state vector

$$\mathbf{x} = \begin{bmatrix} v_C & i_L \end{bmatrix}^\top . \tag{2}$$

d. For a constant input $V_S(t) = 5\,\text{V}$, derive the equilibrium state.
e. Linearize the state-space equation about the operating point

$$\mathbf{x}_o, \mathbf{u}_o = \begin{bmatrix} 5\,\text{V} & 0\,\text{A} \end{bmatrix}^\top, \begin{bmatrix} 5\,\text{V} \end{bmatrix}. \tag{3}$$

Define the state equation matrices A and B, the linearized state and input vectors $\mathbf{x}^*$ and $\mathbf{u}^*$, and the linearized state equation.

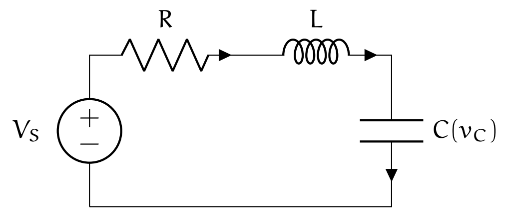

Figure exe.1: circuit for Exercise 14.1 nlin..

Exercise 14.2 franz

A nonlinear diode model gives a diode's elemental equation to be

$$i_D = I_s(\exp(v_D/V_{TH}) - 1).$$

We let the saturation current be $I_s = 10^{-12}$ A and the thermal voltage be $V_{TH} = 0.025$ V. Considering this nonlinear diode model for the circuit of Fig. exe.2.

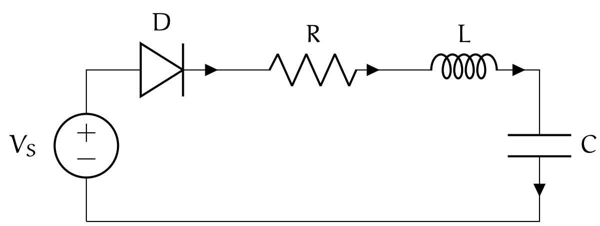

Figure exe.2: circuit for Exercise 14.2 nlin..

a. Derive a nonlinear state-space equation with state vector

$$x = \begin{bmatrix} v_C & i_L \end{bmatrix}^\top. \tag{4}$$

Hint: include the diode in your normal tree.

b. For a constant input $V_S(t) = 0$ V, derive the equilibrium state.

c. Linearize the state-space equation about the operating point

$$x_o, u_o = \begin{bmatrix} 0\,\text{V} & 0\,\text{A} \end{bmatrix}^\top, \begin{bmatrix} 0\,\text{V} \end{bmatrix}. \tag{5}$$

Hint: $d\ln(x)/\,dx = 1/x$. Define the state equation matrices A and B, the linearized state and input vectors x^* and u^*, and the linearized state equation.

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