01.3 math.matrix Matrix inverses

This is a guide to inverting 1 \times 1, 2 \times 2, and n \times n matrices. Let A be the 1 \times 1 matrix

$$\mathsf{A}=\left[\mathfrak{a}\right].$$

The inverse is simply the reciprocal:

$$A^{-1} = \left[1/\alpha\right].$$

Let B be the 2×2 matrix

$$\mathbf{B} = \begin{bmatrix} \mathbf{b}_{11} & \mathbf{b}_{12} \\ \mathbf{b}_{21} & \mathbf{b}_{22} \end{bmatrix}.$$

It can be shown that the inverse follows a simple pattern:

$$B^{-1} = \frac{1}{\det B} \begin{bmatrix} b_{22} & -b_{12} \\ -b_{21} & b_{11} \end{bmatrix}$$
$$= \frac{1}{b_{11}b_{22} - b_{12}b_{21}} \begin{bmatrix} b_{22} & -b_{12} \\ -b_{21} & b_{11} \end{bmatrix}.$$

Let C be an $n \times n$ matrix. It can be shown that its inverse is

$$C^{-1} = \frac{1}{\det C} \operatorname{adj} C,$$

where adj is the **adjoint** of C.