

## 03.4 sum.ft Fourier transforms

Table ft.1 is a table with functions of time  $f(t)$  on the left and corresponding Fourier transforms  $F(\omega)$  on the right. Where applicable,  $T$  is the time-domain period,  $\omega_0 2\pi/T$  is the corresponding angular frequency,  $j = \sqrt{-1}$ ,  $a \in \mathbb{R}^+$ , and  $b, t_0 \in \mathbb{R}$  are constants. Furthermore,  $f_e$  and  $f_o$  are even and odd functions of time, respectively, and it can be shown that any function  $f$  can be written as the sum  $f(t) = f_e(t) + f_o(t)$ . (Hsu, 1967)

**Table ft.1:** Fourier transform identities.

function of time $t$	function of frequency $\omega$
$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(\omega) + a_2 F_2(\omega)$
$f(at)$	$\frac{1}{ a } F(\omega/a)$
$f(-t)$	$F(-\omega)$
$f(t - t_0)$	$F(\omega)e^{-j\omega t_0}$
$f(t) \cos \omega_0 t$	$\frac{1}{2}F(\omega - \omega_0) + \frac{1}{2}F(\omega + \omega_0)$
$f(t) \sin \omega_0 t$	$\frac{1}{j2}F(\omega - \omega_0) - \frac{1}{j2}F(\omega + \omega_0)$
$f_e(t)$	$\text{Re } F(\omega)$
$f_o(t)$	$j \text{Im } F(\omega)$
$F(t)$	$2\pi f(-\omega)$
$f'(t)$	$j\omega F(\omega)$

$\frac{d^n f(t)}{dt^n}$	$(j\omega)^n F(\omega)$
$\int_{-\infty}^t f(\tau) d\tau$	$\frac{1}{j\omega} F(\omega) + \pi F(0)\delta(\omega)$
$-jtf(t)$	$F'(\omega)$
$(-jt)^n f(t)$	$\frac{d^n F(\omega)}{d\omega^n}$
$f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau) f_2(t - \tau) d\tau$	$F_1(\omega) F_2(\omega)$
$f_1(t) f_2(t)$	$\frac{1}{2\pi} F_1(\omega) * F_2(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F_1(\alpha) F_2(\omega - \alpha) d\alpha$
$e^{-at} u_s(t)$	$\frac{1}{j\omega + a}$
$e^{-a t }$	$\frac{2a}{a^2 + \omega^2}$
$e^{-at^2}$	$\sqrt{\pi/a} e^{-\omega^2/(4a)}$
1 for $ t  < a/2$ , else 0	$\frac{a \sin(a\omega/2)}{a\omega/2}$
$te^{-at} u_s(t)$	$\frac{1}{(a + j\omega)^2}$
$\frac{t^{n-1}}{(n-1)!} e^{-at} u_s(t)$	$\frac{1}{(a + j\omega)^n}$
$\frac{1}{a^2 + t^2}$	$\frac{\pi}{a} e^{-a \omega }$
$\delta(t)$	1
$\delta(t - t_0)$	$e^{-j\omega t_0}$
$u_s(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$

$u_s(t - t_0)$	$\pi\delta(\omega) + \frac{1}{j\omega} e^{-j\omega t_0}$
1	$2\pi\delta(\omega)$
t	$2\pi j\delta'(\omega)$
$t^n$	$2\pi j^n \frac{d^n \delta(\omega)}{d\omega^n}$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
$\cos \omega_0 t$	$\pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$
$\sin \omega_0 t$	$-j\pi\delta(\omega - \omega_0) + j\pi\delta(\omega + \omega_0)$
$u_s(t) \cos \omega_0 t$	$\frac{j\omega}{\omega_0^2 - \omega^2} + \frac{\pi}{2}\delta(\omega - \omega_0) + \frac{\pi}{2}\delta(\omega + \omega_0)$
$u_s(t) \sin \omega_0 t$	$\frac{\omega_0}{\omega_0^2 - \omega^2} + \frac{\pi}{2j}\delta(\omega - \omega_0) - \frac{\pi}{2j}\delta(\omega + \omega_0)$
$tu_s(t)$	$j\pi\delta'(\omega) - 1/\omega^2$
1/t	$\pi j - 2\pi j u_s(\omega)$
1/t <sup>n</sup>	$\frac{(-j\omega)^{n-1}}{(n-1)!} (\pi j - 2\pi j u_s(\omega))$
sgn t	$\frac{2}{j\omega}$
$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\omega_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$

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