1.2 Voltage Dividers

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In chapter 2 we'll learn about how to approach circuit analysis in a systematic way. For now, we'll limp along unsystematically with our toolbelt of concepts and equations in order to introduce some more circuit elements, concepts, and theorems. But we can't resist just a bit of circuit analysis now.

The **voltage divider** is a ubiquitous and useful circuit. In a sense, it's less of a circuit and more of concept. For resistors, that concept can be stated as the following.

> The voltage across resistors in series is divided among the resistors.

An immediately useful result is that we can "divide voltage" into any smaller voltage we like by putting in a couple resistors.

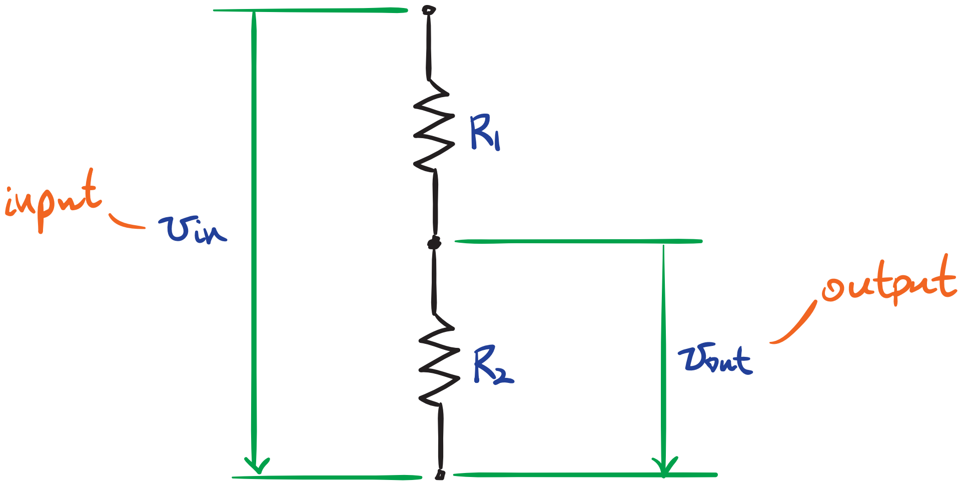

Figure 1.1. a simple voltage divider circuit.

In order to show *how* the voltage divider "divides up" the voltage, we must do some basic circuit analysis. Consider the circuit in figure 1.1. The input voltage v_{in} is divided into v_{R_1} and $v_{R_2} = v_{\text{out}}$. We want to know v_{out} as a function of v_{in} and parameters R_1 and R_2. Let's write down the equations we know from the laws of Kirchhoff and Ohm:

$$
\begin{aligned}
v_{R_1} &= i_{R_1} R_1, \\
v_{R_2} &= i_{R_2} R_2, \\
v_{\text{in}} &= v_{R_1} + v_{R_2}, \text{ and} \\
i_{R_1} &= i_{R_2}.
\end{aligned}
$$

We've already established that $v_{\mathrm{out}} = v_{R_2}$, so we can solve for v_{R_2} in (∗). We want to eliminate the three "unknown" variables v_{R_1}, i_{R_1}, and i_{R_2}, so it is good that we have four equations.[2] We begin with (∗b) and proceed by substitution of the others of (∗):

$$\begin{aligned}
v_{R_2} &= i_{R_2} R_2 \\
&= i_{R_1} R_2 \\
&= \frac{R_2}{R_1} v_{R_1} \\
&= \frac{R_2}{R_1} \left(v_{\mathrm{in}} - v_{R_2} \right) \implies \\
v_{R_2} + \frac{R_2}{R_1} v_{R_2} &= \frac{R_2}{R_1} v_{\mathrm{in}} \implies \\
v_{R_2} &= \frac{R_2/R_1}{1 + R_2/R_1} v_{\mathrm{in}} \implies \\
&= \frac{R_2}{R_1 + R_2} v_{\mathrm{in}}.
\end{aligned}$$

Nice! So we can now write the input-output relationship for a two-resistor voltage divider.

Definition 1.9

For a voltage v_{in} split across two resistors R_1 and R_2 in series, the voltage divider equation is

$$v_{R_2} = \frac{R_2}{R_1 + R2} v_{\mathrm{in}}$$

for R_2 or

$$v_{R_1} = \frac{R_1}{R_1 + R2} v_{\mathrm{in}}$$

for R_1.

So the voltage divider had the effect of dividing the input voltage into a fraction governed by the relationship between the relative resistances of the two resistors. This fraction takes values in the interval $[0, 1]$. Now, whenever we see the voltage divider circuit, we can just remember this easy formula!

Similarly, for n resistors in series, it can be shown that the voltage divider relationship is as follows.

2. Alternatively, we could solve for all four unknown variables with our four equations.

Definition 1.10

For a voltage v_{in} split across n resistors $R_1, R_2, \cdots, R_n$ in series, the voltage divider equation is

$$v_{R_m} = \left(R_m \middle/ \sum_{k=1}^{n} R_k \right) v_{\text{in}}$$

for resistor R_m.

1.3 Sources

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Sources (i.e., supplies) supply power to a circuit. There are two primary types: *voltage sources* and *current sources*.

1.3.1 Ideal Voltage Sources

An ideal voltage source provides exactly the voltage a user specifies, independent of the circuit to which it is connected. All it must do in order to achieve this is to supply whatever current necessary. Let's unpack this with a simple example.

Example 1.2

In the circuit shown, determine how much current and power the ideal voltage source V_s must provide in order to maintain voltage if $R \to \infty$ and if $R \to 0$.

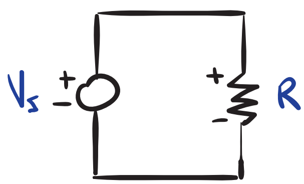

Figure 1.2. A circuit with a voltage source connected to a resistor load.

Since the voltage across the resistor is known to be equal to V_s, Ohm's law tells us that

$$i_R = V_s / R.$$

Of course, power dissipated by the resistor is

$$\begin{aligned} \mathcal{P}_R &= i_R v_R \\ &= V_s^2 / R. \end{aligned}$$