Definition 1.10

For a voltage v_{in} split across n resistors $R_1, R_2, \cdots, R_n$ in series, the voltage divider equation is

$$v_{R_m} = \left(R_m \middle/ \sum_{k=1}^{n} R_k \right) v_{\text{in}}$$

for resistor R_m.

1.3 Sources

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Sources (i.e., supplies) supply power to a circuit. There are two primary types: *voltage sources* and *current sources*.

1.3.1 Ideal Voltage Sources

An ideal voltage source provides exactly the voltage a user specifies, independent of the circuit to which it is connected. All it must do in order to achieve this is to supply whatever current necessary. Let's unpack this with a simple example.

Example 1.2

In the circuit shown, determine how much current and power the ideal voltage source V_s must provide in order to maintain voltage if $R \to \infty$ and if $R \to 0$.

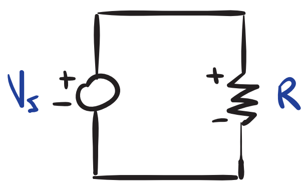


Figure 1.2. A circuit with a voltage source connected to a resistor load.

Since the voltage across the resistor is known to be equal to V_s, Ohm's law tells us that

$$i_R = V_s / R.$$

Of course, power dissipated by the resistor is

$$\begin{aligned} \mathcal{P}_R &= i_R v_R \\ &= V_s^2 / R. \end{aligned}$$

Taking the "open circuit" limit,

$$i_{R\to\infty} = \lim_{R\to\infty} V_s/R$$
$$= 0 \text{ A}$$
$$\mathcal{P}_{R\to\infty} = \lim_{R\to\infty} V_s^2/R$$
$$= 0 \text{ W}.$$

Taking the "short circuit" limit,

$$i_{R\to 0} = \lim_{R\to 0} V_s/R$$
$$\to \infty \text{ A}$$
$$\mathcal{P}_{R\to 0} = \lim_{R\to 0} V_s^2/R$$
$$\to \infty \text{ W}.$$

1.3.2 Ideal Current Sources

An ideal current source provides exactly the current a user specifies, independent of the circuit to which it is connected. All it must do in order to achieve this is to supply whatever voltage necessary. Let's unpack this with a simple example.

Example 1.3

In the circuit shown, determine how much voltage and power the ideal current source I_s must provide in order to maintain voltage if $R \to 0$ and if $R \to \infty$.

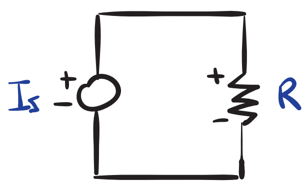


Figure 1.3. A circuit with a current source connected to a resistor load.

Since the current through the resistor is known to be equal to I_s, Ohm's law tells us that

$$v_R = I_s R.$$

Of course, power dissipated by the resistor is

$$\begin{aligned}\mathcal{P}_R &= i_R v_R \\ &= I_s^2 R.\end{aligned}$$

Taking the "short circuit" limit,

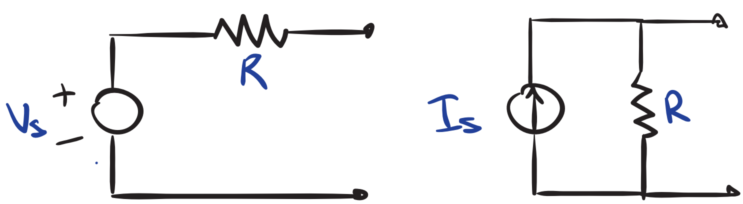
$$\begin{aligned}v_{R\to 0} &= \lim_{R\to 0} I_s R \\ &\to 0\text{ A} \\ \mathcal{P}_{R\to 0} &= \lim_{R\to 0} I_s^2 R \\ &\to 0\text{ W.}\end{aligned}$$

Taking the "open circuit" limit,

$$\begin{aligned}v_{R\to \infty} &= \lim_{R\to \infty} I_s R \\ &= \infty\text{ A} \\ \mathcal{P}_{R\to \infty} &= \lim_{R\to \infty} I_s^2 R \\ &= \infty\text{ W.}\end{aligned}$$

1.3.3 Modeling Real Sources

No real source can produce infinite power. Some have feedback that controls the output within some finite power range. These types of sources can be approximated as ideal when operating within their specifications. Many voltage sources (e.g. batteries) do not have internal feedback controlling the voltage. When these sources are "loaded" (delivering power) they cannot maintain their nominal output, be that voltage or current. We model these types of sources as ideal sources in series or parallel with a resistor, as illustrated in figure 1.4.



(a) Real voltage source model (b) Real current source model.

Figure 1.4. Models for power-limited "real" sources.

Most manufacturers specify the nominal resistance of a source as the "output resistance." A typical value is 50 Ω.

1.4 Thévenin's and Norton's Theorems

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Thévenin's and Norton's theorems yield ways to simplify our models of circuits.

1.4.1 Thévenin's Theorem

The following remarkable theorem has been proven.

Theorem 1.11

Thévenin's theorem: Given a linear network of voltage sources, current sources, and resistors, the behavior at the network's output terminals can be reproduced exactly by a single *voltage source* V_e *in series with a resistor* R_e.

The equivalent circuit has two quantities to determine: V_e and R_e.

1.4.1.1 Determining R_e The **equivalent resistance** R_e of a circuit is the resistance between the output terminals with all inputs set to zero. Setting a voltage source to zero means the voltage on both its terminals are equal, which is equivalent to treating it as a short or wire. Setting a current source to zero means the current through it is zero, which is equivalent to treating it as an open circuit.

1.4.1.2 Determining V_e The **equivalent voltage source** V_e is the voltage at the output terminals of the circuit when they are left open (disconnected from a load). Determining this value typically requires some circuit analysis with the laws of Ohm and Kirchhoff.

1.4.2 Norton's Theorem

Similarly, the following remarkable theorem has been proven.

Theorem 1.12

Norton's theorem: Given a linear network of voltage sources, current sources, and resistors, the behavior at the network's output terminals can be reproduced exactly by a single *current source* I_e *in parallel with a resistor* R_e.

The equivalent circuit has two quantities to determine: I_e and R_e. The equivalent resistance R_e is identical to that of Thévenin's theorem, which leaves the equivalent current source I_e to be determined.