2 Circuit Analysis

In this chapter, we consider the techniques of analog circuit analysis. We use a time-domain approach in which the dynamics of circuits are expressed as ordinary differential equations (ODEs).

2.1 Sign Convention

We use the **passive sign convention** of electrical engineering, defined below and illustrated in figure 2.1.

Definition 2.1

Power flowing *in* to a component is considered to be *positive* and power flowing *out* of a component is considered *negative*.

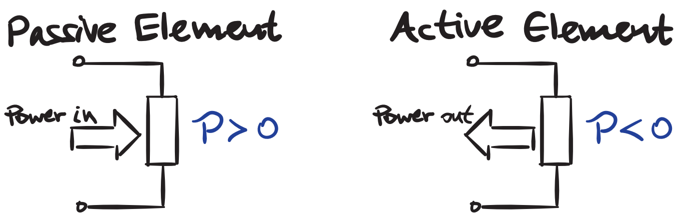

Figure 2.1. passive sign convention in terms of power $\mathcal{P}$.

Because power $\mathcal{P} = vi$, this implies the current and voltage signs are prescribed by the convention. For **passive elements**, the electrical potential must drop in the direction of positive current flow. This means the assumed direction of voltage drop across a passive element must be the same as that of the current flow. For **active elements**, which supply power to the circuit, the converse is true: the voltage drop and current flow must be in opposite directions. figure 2.2 illustrates the possible configurations.

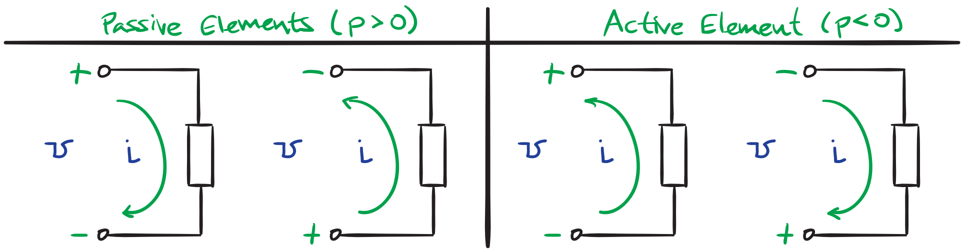

Figure 2.2. passive sign convention in terms of voltage v and current i.

When analyzing a circuit, for each passive element, draw an arrow beside it pointing in the direction of assumed current flow and voltage drop. Try it out on figure 2.3.

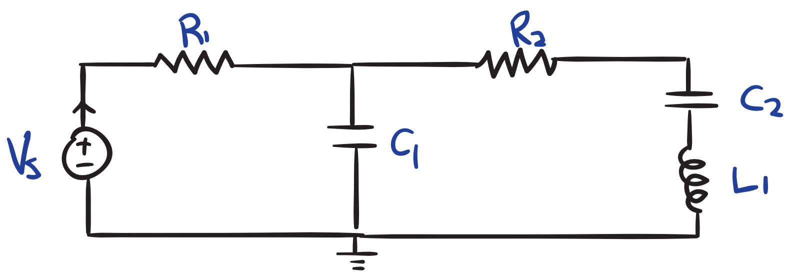

Figure 2.3. an illustration of the passive sign convention on a circuit.

The purpose of a sign convention is to help us **interpret** the signs of our results. For instance, if, at a given instant, a capacitor has voltage $v_C = 3$ V and current $i_C = -2$ A, we compute $\mathcal{P}_C = -6$ W and we know 6 W of power is flowing *from* the capacitor into the circuit.

For passive elements, there is no preferred direction of "assumed" voltage drop and current flow. If a voltage or current value discovered by performing a circuit analysis is positive, this means the "assumed" and "actual" directions are the same. For a negative value, the directions are opposite.

For active elements, *we don't get to choose* the direction. The physical situation prescribes it. For instance, if a +12 V positive terminal of a battery is connected to a certain terminal in a circuit, the voltage of that terminal should be assigned a positive voltage of +12 V.

2.2 Methodology for Analyzing Circuits

We have all the tools we need to do some pretty badass circuit analysis. Later we'll learn a more systematic method for analyzing the dynamics of a circuit, but for now we can use broad strokes to get the idea. It will work most of the time, but occasionally you may need to write some extra KCL or KVL equations or use a more advanced algebraic technique.

Let n be the number of passive circuit elements in a circuit, which gives $2n$ (v and i for each element) unknowns. The method is this.

1. Draw a *circuit diagram*.
2. Label the circuit diagram with the *sign assignment* by labeling each element with the "assumed" direction of current flow.
3. Write the *elemental equation* for each circuit element (e.g. Ohm's law).
4. For every node not connected to a voltage source, write Kirchhoff's current law (KCL).
5. For each loop not containing a current source, write Kirchhoff's voltage law (KVL).
6. You probably have a linear system of $2n$ algebraic and first-order, ordinary differential equations (and $2n$ unknowns) to be solved simultaneously.
    1. Eliminate n (half) of the unknowns by substitution into the elemental equations.
    2. Try substition or elimination to get down to only those variables with time derivatives and inputs. If this doesn't work, use a linear algebra technique.
    3. Solve the remaining set of first-order, linear ordinary differential equations. This can be done either directly or by turning it into a single higher-order differential equation and then solving.

Example 2.1

In the RC circuit shown, let $V_s(t) = 12$ V. If $v_C(t)|_{t=0} = 0$, what is $v_o(t)$ for $t \geq 0$?.