to a certain terminal in a circuit, the voltage of that terminal should be assigned a positive voltage of +12 V.

2.2 Methodology for Analyzing Circuits

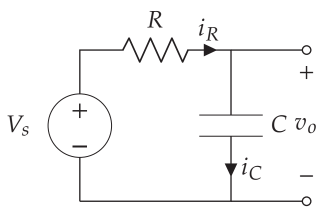
We have all the tools we need to do some pretty badass circuit analysis. Later we'll learn a more systematic method for analyzing the dynamics of a circuit, but for now we can use broad strokes to get the idea. It will work most of the time, but occasionally you may need to write some extra KCL or KVL equations or use a more advanced algebraic technique.

Let n be the number of passive circuit elements in a circuit, which gives $2n$ (v and i for each element) unknowns. The method is this.

1. Draw a *circuit diagram*.
2. Label the circuit diagram with the *sign assignment* by labeling each element with the "assumed" direction of current flow.
3. Write the *elemental equation* for each circuit element (e.g. Ohm's law).
4. For every node not connected to a voltage source, write Kirchhoff's current law (KCL).
5. For each loop not containing a current source, write Kirchhoff's voltage law (KVL).
6. You probably have a linear system of $2n$ algebraic and first-order, ordinary differential equations (and $2n$ unknowns) to be solved simultaneously.
    1. Eliminate n (half) of the unknowns by substitution into the elemental equations.
    2. Try substition or elimination to get down to only those variables with time derivatives and inputs. If this doesn't work, use a linear algebra technique.
    3. Solve the remaining set of first-order, linear ordinary differential equations. This can be done either directly or by turning it into a single higher-order differential equation and then solving.

Example 2.1

In the RC circuit shown, let $V_s(t) = 12$ V. If $v_C(t)|_{t=0} = 0$, what is $v_o(t)$ for $t \geq 0$?.



1. The circuit diagram is given.
2. The signs are given.
3. The n elemental equations are as follows.

C	$\dfrac{dv_C}{dt} = \dfrac{1}{C} i_C$
R	$v_R = i_R R$

4. There is one node not connected to the voltage source, for which the KCL equation is
$$i_C = i_R.$$
5. There is one loop which has KVL equation is
$$v_R + v_C - V_s = 0.$$
6. Solve.
   1. Eliminate i_C and v_R using KCL and KVL to yield the following.

C	$\frac{dv_C}{dt} = \frac{1}{C} i_R$
R	$i_R = \frac{1}{R}(V_s - v_C)$

   2. Substituting the R equation into the C equation, we eliminate i_R:
$$\frac{dv_C}{dt} = \frac{1}{RC}(V_s - v_C).$$
   3. Rearranging,
$$RC\frac{dv_C}{dt} + v_C = V_s.$$
   Let's solve the ODE with initial condition $v_C(0) = 0$.
      1. Homogeneous solution. The characterestic equation is
$$RC\lambda + 1 = 0 \quad \Rightarrow \quad \lambda = -\frac{1}{RC}.$$
      Therefore, $v_{Ch}(t) = k_1 e^{-t/(RC)}$.
      2. Particular solution. The form of the input $V_s(t)$ invites us to assume $v_{Cp}(t) = k_2$. Substituting into the ODE,
$$RC(0) + k_2 = 12 \text{ V} \quad \Rightarrow \quad k_2 = 12 \text{ V}.$$
      3. Total solution: $v_C(t) = v_{Ch}(t) + v_{Cp}(t)$.
      4. Specific solution:
$$v_C(0) = 0 \quad \Rightarrow \quad k_1 + k_2 = 0 \quad \Rightarrow \quad k_1 = -k_2 = -12\text{V}.$$
      Since, $v_o(t) = v_C(t)$,
$$v_o(t) = 12(1 - e^{-t/(RC)}).$$

2.3 A Sinusoidal Input Example