2.4 Transient and Steady-State Response

Let's consider them response of the circuit in example 2.2. We found that the inductor had current and voltage responses

$$i_L(t) = \left(i_0 + \frac{A\tau\omega}{(\tau\omega)^2+1}\right)e^{-t/\tau} + \frac{A}{\sqrt{(\tau\omega)^2+1}}\sin(\omega t - \arctan \tau\omega)$$

and

$$v_L(t) = -\frac{L}{\tau}\left(i_0 + \frac{A\tau\omega}{(\tau\omega)^2+1}\right)e^{-t/\tau} + \frac{AL\omega}{\sqrt{(\tau\omega)^2+1}}\cos(\omega t - \arctan \tau\omega).$$

Note that the top line of each of these equations decays exponentially to zero. The response while this term dominates is the *transient response* and the response thereafter is the *steady-state response*.

In 6τ (six time constants) the exponential term has decayed to less than 1 %, so we often assume the other term will be dominating by that point.

We will plot $i_L(t)$ and $v_L(t)$ from above to illustrate transient and steady-state response.

Let's use Python to plot the response. We begin by loading two packages as follows:

```
import numpy as np  # For numerical calculations
import matplotlib.pyplot as plt  # For plotting
```

Plots cannot be created without some definition of parameters. Let us define them as follows:

```
R = 1  # (Ohms) resistance
L = 1e-3  # (H) inductance
i_0 = 10  # (A) initial current in inductor
A = 10  # (A) sinusoidal input amplitude
omega = 5e3  # (rad/s) sinusoidal input angular frequency
tau = L/R  # (s) time constant
```

The current and voltage can be defined as follows:

```
def i_L(t):
    return (i_0+A*tau*omega/((tau*omega)^2+1))*exp(-t/tau) + \
        (A/sqrt((tau*omega)^2+1)* \
        sin(omega*t - arctan2(tau*omega,1)))
def v_L(t):
    return -L/tau*(i_0+A*tau*omega/ \
    ((tau*omega)^2+1))*exp(-t/tau) + \
    (A*L*omega/sqrt((tau*omega)^2+1)* \
     cos(omega*t - arctan2(tau*omega,1)))
```

Now we turn to defining simulation parameters.

```
N = 201  # Number of points to plot
t_min = 0  # Minimum time
t_max = 8 * tau  # Maximum time
t_s = np.linspace(t_min, t_max, N)  # Array of time values
```

Now to create numerical arrays to plot.

```
i_Ls = [] # initializing sampled array
v_Ls = [] # initializing sampled array
for i in range(0, N):
    i_Ls.append(i_L(t_s[i])) # build array of values
    v_Ls.append(v_L(t_s[i])) # build array of values
```

We use the package `matplotlib` to plot the inductor voltage and current.

```
fig = plt.figure()
ax = plt.subplot(111)
ax.plot(t_s, i_Ls, 'b-', linewidth=2, label='$i_L(t)$')  # plot
ax.plot(t_s, v_Ls, 'r-', linewidth=2, label='$v_L(t)$')  # plot
ax.legend(loc='center left', bbox_to_anchor=(1, 0.5))
ax.set_xlabel('time (s)')
plt.show()  # Display
```

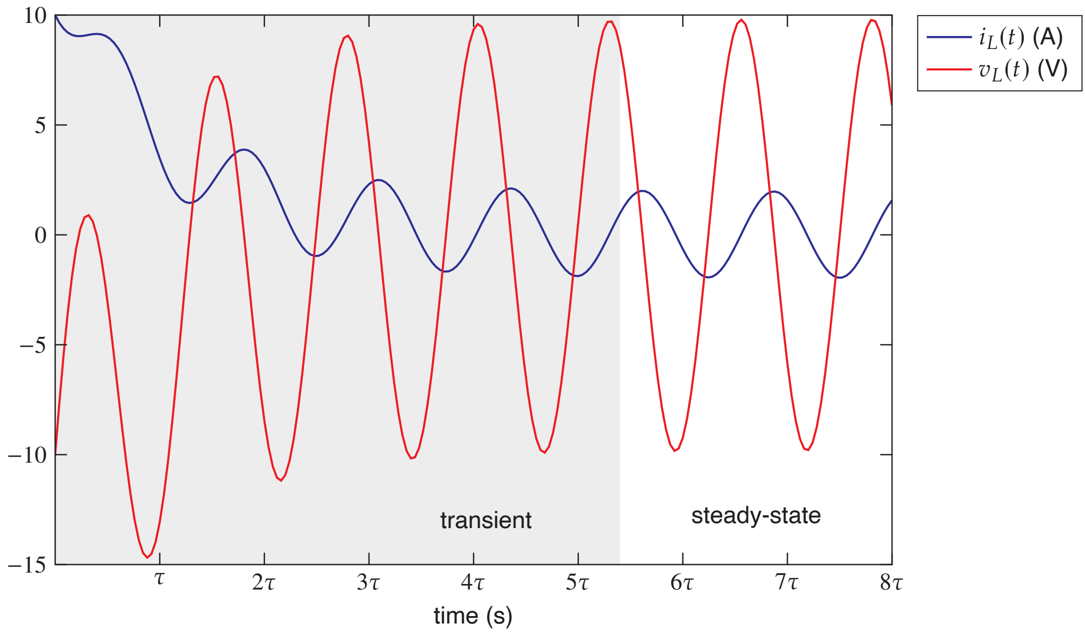


Figure 2.4. Current i_L and voltage v_L of the inductor for transient and steady-state response. Note that the transition is not precisely defined.

Figure 2.4 shows that in around six time constants, as is typical, the responses settle in to steady oscillations. Note that the steady-state is not necessarily *static*, but can also be oscillatory, as in this case. In fact, every linear dynamic system driven by a sinusoid will have a sinusoidal steady-state response, as we will explore further in the coming lectures.

Often the term *ac circuit analysis* is used refer to circuits with sinusoidal sources *in steady-state*. In many circuits, steady-state is acheived relatively quickly, which is why this is the most popular type of analysis. Our approach has yielded *both* responses, together. In order to consider the steady-state only, all we must do is ignore the exponentially decaying terms, which are the initial conditions' contributions to the transient response.

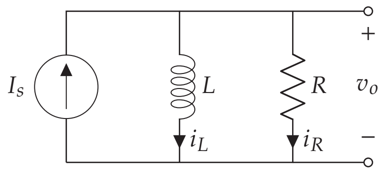
However, there are easier methods of obtaining the steady-state response if the transient response isn't of interest. The next chapter (chapter 3) considers these.

2.5 Problems

OX

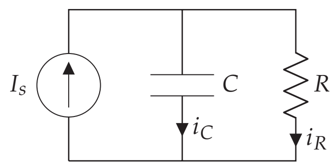
Problem 2.1 MAD Use the diagram below to answer the following questions and imperatives. Let $I_s = A_0$, where $A_0 \in \mathbb{R}$ is a known constant. Perform a full circuit analysis, including the transient response. The initial inductor current is $i_L(0) = 0$.

(a) Write the elemental, KCL, and KVL equations.
(b) Write the differential equation for $i_L(t)$ arranged in the standard form and identify the time constant τ.
(c) Solve the differential equation for $i_L(t)$ and use the solution to find the output voltage $v_o(t)$.



Problem 2.2 THEOCRATICALLY Use the diagram below to answer the following questions and imperatives. Let $I_s = A_0$, where $A_0 \in \mathbb{R}$ is a known constant. Perform a full circuit analysis, including the transient response. The initial capacitor voltage is $v_C(0) = v_{C0}$, a known constant.

(a) Write the elemental, KCL, and KVL equations.
(b) Write the differential equation for $v_C(t)$ arranged in the standard form.
(c) Solve the differential equation for $v_C(t)$.



Problem 2.3 HIPPOPHOBIA For the RC circuit diagram below, perform a complete circuit analysis to solve for $v_o(t)$ if $V_S(t) = A \sin \omega t$, where $A \in \mathbb{R}$ is a given amplitude and $\omega \in \mathbb{R}$ is a given angular frequency. Let $v_C(t)|_{t=0} = v_{C0}$, where $v_{C0} \in \mathbb{R}$ is a given initial capacitor voltage. Hint: you will need to solve a differential equation for $v_C(t)$.