2.5 Problems

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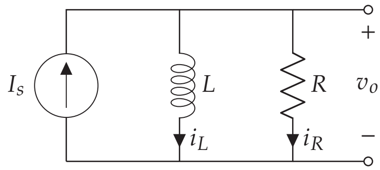
Problem 2.1 MAD Use the diagram below to answer the following questions and imperatives. Let $I_s = A_0$, where $A_0 \in \mathbb{R}$ is a known constant. Perform a full circuit analysis, including the transient response. The initial inductor current is $i_L(0) = 0$.

(a) Write the elemental, KCL, and KVL equations.

(b) Write the differential equation for $i_L(t)$ arranged in the standard form and identify the time constant τ.

(c) Solve the differential equation for $i_L(t)$ and use the solution to find the output voltage $v_o(t)$.

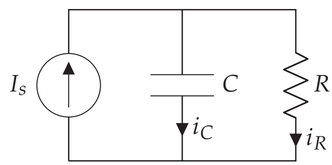


Problem 2.2 THEOCRATICALLY Use the diagram below to answer the following questions and imperatives. Let $I_s = A_0$, where $A_0 \in \mathbb{R}$ is a known constant. Perform a full circuit analysis, including the transient response. The initial capacitor voltage is $v_C(0) = v_{C0}$, a known constant.

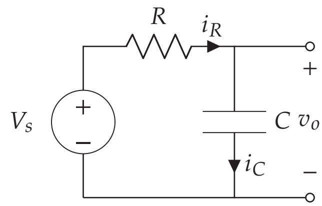
(a) Write the elemental, KCL, and KVL equations.

(b) Write the differential equation for $v_C(t)$ arranged in the standard form.

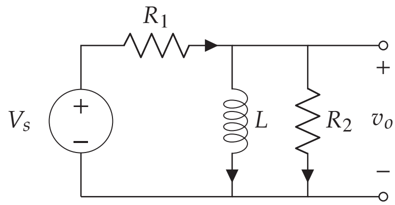
(c) Solve the differential equation for $v_C(t)$.



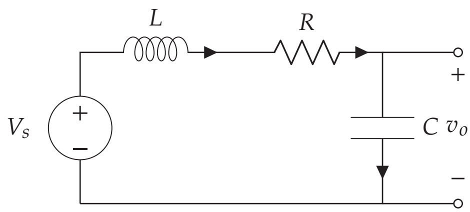
Problem 2.3 HIPPOPHOBIA For the RC circuit diagram below, perform a complete circuit analysis to solve for $v_o(t)$ if $V_S(t) = A \sin \omega t$, where $A \in \mathbb{R}$ is a given amplitude and $\omega \in \mathbb{R}$ is a given angular frequency. Let $v_C(t)|_{t=0} = v_{C0}$, where $v_{C0} \in \mathbb{R}$ is a given initial capacitor voltage. Hint: you will need to solve a differential equation for $v_C(t)$.



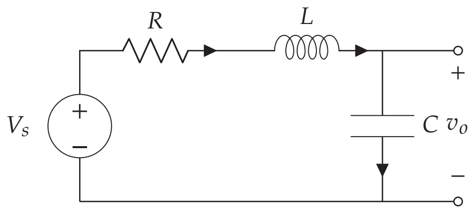
Problem 2.4 FRUITARIANISM For the circuit diagram below, perform a complete circuit analysis to solve for $v_o(t)$ if $V_s(t) = A \sin \omega t$, where $A \in \mathbb{R}$ is a given amplitude and $\omega \in \mathbb{R}$ is a given angular frequency. Let $i_L(t)|_{t=0} = 0$ be the initial inductor current. Hint: you will need to solve a differential equation for $i_L(t)$.



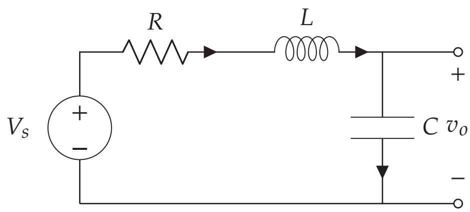
Problem 2.5 GASTROLOBIUM For the circuit diagram below, perform a complete circuit analysis to solve for $v_o(t)$ if $V_s(t) = 0$. Let $v_C(t)|_{t=0} = 5$ V and $dv_C/dt|_{t=0} = 0$ V/s be the initial conditions. Assume the characteristic equation has distinct roots. Recommendation: due to the initial conditions being given in it, solve the differential equation in v_C.



Problem 2.6 THYROPRIVIC For the circuit diagram below, perform a complete circuit analysis to solve for $v_o(t)$ if $V_s(t) = 3 \sin(10t)$. Let $v_C(t)|_{t=0} = 0$ V and $dv_C/dt|_{t=0} = 0$ V/s be the initial conditions. Assume the characteristic equation has distinct, complex roots. Recommendation: due to the initial conditions being given in it, solve the differential equation in v_C. Also, consider which, if any, of your results from problem 2.5 apply and re-use them, if so.

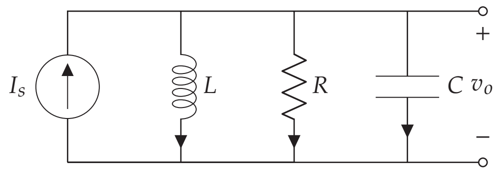


Problem 2.7 HEMOGENESIS For the circuit diagram below, solve for $v_o(t)$ if $V_s(t) = A\ \sin \omega t$, where $A = 2$ V is the given amplitude and $\omega \in \mathbb{R}$ is a given angular frequency. Let $R = 50\ \Omega$, $L = 50$ mH, and $C = 200$ nF. Let the circuit have initial conditions $v_C(0) = 1$ V and $i_L(0) = 0$ A. Find the steady-state ratio of the output amplitude to the input amplitude A for $\omega = \{5000, 10000, 50000\}$ rad/s. This circuit is called a **low-pass filter**—explain why this makes sense. Plot $v_o(t)$ in MATLAB, Python, or Mathematica for $\omega = 400$ rad/s (you think this won't be part of the quiz, but it will be!). Hint: either re-write your system of differential-algebraic equations and initial conditions as a single second-order differential equation with initial conditions in the differential variable *or* re-write it as a system of two first-order differential equations and solve that.



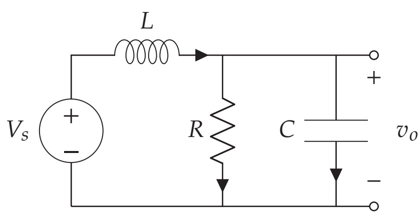
Problem 2.8 PHOTOCHROMASCOPE Use the circuit diagram below to answer the following questions and imperatives. Let $I_s = A_0$, where $A_0 > 0$ is a known constant. Perform a full circuit analysis, including the transient response. The initial inductor current is $i_L(0) = 0$ and the initial capacitor voltage is $v_C(0) = 0$. Assume the damping ratio $\zeta \in (0, 1)$; i.e. the system is underdamped and the roots of the characteristic equation are complex.

(a) Write the elemental, KCL, and KVL equations.
(b) Write the second-order differential equation for $i_L(t)$ arranged in the standard form and identify the natural frequency ω_n and damping ratio ζ.
(c) Convert the initial condition in v_C to a *second* initial condition in i_L.
(d) Solve the differential equation for $i_L(t)$ and use the solution to find the output voltage $v_o(t)$. It is acceptable to use a known solution and to express your solution in terms of ω_n and ζ.



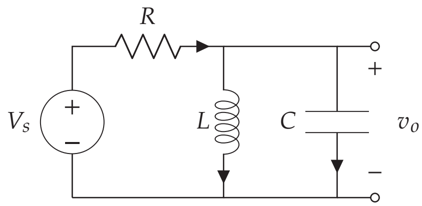
Problem 2.9 STADIUM Use the circuit diagram below to answer the following questions and imperatives. Let $V_s = A_0$, where $A_0 > 0$ is a known constant. The initial capacitor voltage is $v_C(0) = 0$ and the initial inductor current is $i_L(0) = 0$.

(a) Write the elemental, KCL, and KVL equations.
(b) Derive the second-order differential equation in $v_C(t)$.
(c) Write the differential equation in standard form and identify the natural frequency ω_n and damping ratio ζ.
(d) Convert the initial condition in i_L to a *second* initial condition in v_C.
(e) *Do not solve the differential equation*, but outline the steps you would take to solve it.



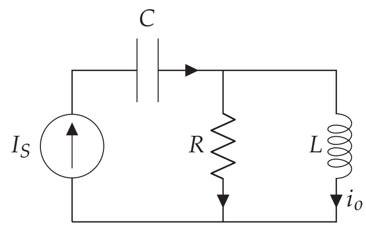
Problem 2.10 MIROT Use the circuit diagram below to answer the following questions and imperatives. Let $V_s = A_0$, where $A_0 > 0$ is a known constant. The initial capacitor voltage is $v_C(0) = 5$ V and the initial inductor current is $i_L(0) = 0$.

(a) Write the elemental, KCL, and KVL equations.
(b) Derive the second-order differential equation in $v_C(t)$.
(c) Write the differential equation in standard form and identify the natural frequency ω_n and damping ratio ζ.
(d) Convert the initial condition in i_L to a *second* initial condition in v_C (i.e., $v_C'(0)$).
(e) *Do not solve the differential equation*, but outline the steps you would take to solve it.



Problem 2.11 UMBRELLA Use the circuit diagram below to answer the following questions and imperatives. Let $I_S = A_0$, where $A_0 > 0$ is a known constant. The initial capacitor voltage is $v_C(0) = 0$ V and the initial inductor current is $i_L(0) = 5$ A.

(a) Write the elemental, KCL, and KVL equations.
(b) Derive the first-order differential equation in $i_L(t)$.
(c) Write the differential equation in standard form with a time constant τ.
(d) Solve the differential equation (find the specific solution) for $i_o(t) = i_L(t)$.



3 Steady-State Circuit Analysis for Sinusoidal Inputs

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Steady-state circuit analysis for sinusoidal inputs does not require the, at times, lengthy process of solving differential equations. Impedance methods, presented in this chapter, are shortcuts to steady-state analysis. It is important to note that impedance methods do not give information about the transient response.

3.1 Complex or Phasor Representations of Voltage and Current

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It is common to represent voltage and current in circuits as complex exponentials, especially when they are sinusoidal. **Euler's formula** is our bridge back-and forth from trigonomentric form ($\cos\theta$ and $\sin\theta$) and exponential form ($e^{j\theta}$):

$$e^{j\theta} = \cos\theta + j\sin\theta.$$

Here are a few useful identities implied by Euler's formula.

$$\begin{aligned} e^{-j\theta} &= \cos\theta - j\sin\theta \\ \cos\theta &= \Re(e^{j\theta}) \\ &= \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right) \\ \sin\theta &= \Im(e^{j\theta}) \\ &= \frac{1}{j2}\left(e^{j\theta} - e^{-j\theta}\right). \end{aligned}$$

These equations can be considered to be describing a vector in the **complex plane**, which is illustrated in figure 3.1. Note that a $e^{j\theta}$ has both a *magnitude* and a *phase*.