3.4 Voltage and Current Dividers

DG

In section 1.2, we developed the useful voltage divider formula for quickly analyzing how voltage divides among series resistors. This can be considered a special case of a more general voltage divider equation for any elements described by an impedance. After developing the voltage divider, we also introduce the current divider, which divides an input current among parallel elements.

3.4.1 Voltage Dividers

First, we develop the solution for the two-element voltage divider shown in figure 3.5. We choose the voltage across Z_2 as the output. The analysis can follow our usual methodology of six steps, solving for v_2.

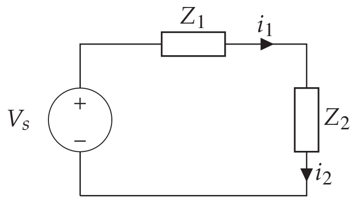


Figure 3.5. The two-element voltage divider.

1. The circuit diagram is given in figure 3.5.
2. The assumed directions of positive current flow are given in figure 3.5.
3. The elemental equations are just generalized Ohm's law equations.

$$\begin{array}{c|l} Z_1 & v_1 = i_1 Z_1 \\ Z_2 & v_2 = i_2 Z_2 \end{array}$$

4. The KCL equation is $i_2 = i_1$.
5. The KVL equation is $v_1 = V_s - v_2$.
6. Solve.
   1. Eliminating i_2 and v_1 from KCL and KVL, our elemental equations become the following.

   $$\begin{array}{c|l} Z_1 & i_1 = v_1/Z_1 = (V_s - v_2)/Z_1 \\ Z_2 & v_2 = i_1 Z_2 \end{array}$$

   2. Eliminating i_1,

   $$v_2 = \frac{Z_2}{Z_1}(V_s - v_2).$$

3. Solving for v_2,

$$v_2 = \frac{Z_2}{Z_1 + Z_2} V_s.$$

A similar analysis can be conducted for n impedance elements.

Definition 3.2: Impedance Voltage Divider

For the output voltage across impedance Z_k in series with n impedance elements with input v_{in} is

$$v_k = \frac{Z_k}{Z_1 + Z_2 + \cdots + Z_k + \cdots + Z_n} v_{\text{in}}.$$

3.4.2 Current Dividers

By a similar process, we can analyze a circuit that divides current into n *parallel* impedance elements.

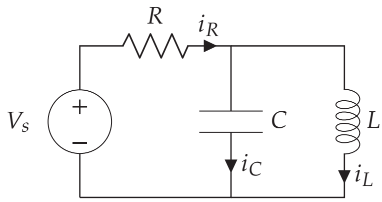
Definition 3.3: Impedance Current Divider

For the output current through impedance Z_k in parallel with n impedance elements with input current i_{in} is

$$i_k = \frac{1/Z_k}{1/Z_1 + 1/Z_2 + \cdots + 1/Z_k + \cdots + 1/Z_n} i_{\text{in}}.$$

Example 3.3

Given the circuit shown with voltage source $V_s(t) = Ae^{j\phi}$ and output v_L, what is the ratio of output over input amplitude? What is the phase shift from input to output?



We'll use a voltage divider:

$$
\begin{aligned}
v_L &= \frac{\frac{Z_L Z_C}{Z_L+Z_C}}{Z_R + \frac{Z_L Z_C}{Z_L+Z_C}} V_s \quad \Rightarrow \\
\frac{v_L}{V_s} &= \frac{Z_L Z_C}{Z_R Z_L + Z_R Z_C + Z_L Z_C} \\
&= \frac{L/C}{jRL\omega - jR/(C\omega) + L/C} \\
&= \frac{L\omega}{jRLC\omega^2 - jR + L\omega} \\
&= \frac{L\omega}{L\omega + jR(LC\omega^2 - 1)} \\
&= \underbrace{\frac{L\omega}{\sqrt{(L\omega)^2 + R^2(LC\omega^2 - 1)^2}}}_{\text{magnitude ratio}} \exp \underbrace{\left(- \arctan \frac{R(LC\omega^2 - 1)}{L\omega} \right)}_{\text{phase difference}}.
\end{aligned}
$$

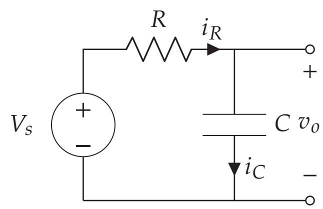
3.5 Problems

SJ

Problem 3.1 NP For the RC circuit diagram below, perform a circuit analysis to solve for the steady state voltage $v_o(t)$ if $V_S(t) = A\ \sin \omega t$, where $A \in \mathbb{R}$ is a given amplitude and $\omega \in \mathbb{R}$ is a given angular frequency. Use a *sine* phasor in the problem. Write your answer as a single sine phasor in *polar* form. Evaluate your answer for the following two sets of parameters.

$$A = 2, 5 \text{ V}$$
$$\omega = 10 \times 10^3, 20 \times 10^3 \text{ rad/s}$$
$$R = 100, 1000\ \Omega$$
$$C = 100, 10 \text{ nF}.$$

The first set should yield $v_o = 1.99e^{-j0.0997}$.



Problem 3.2 ZK For the circuit diagram below, perform a complete circuit analysis to solve for the steady state voltage $v_o(t)$ if $V_s(t) = A\ \sin \omega t$, where $A \in \mathbb{R}$ is a given amplitude and $\omega \in \mathbb{R}$ is a given angular frequency. Use a *sine* phasor in the problem. Write your answer as a single sine phasor in *polar* form. Evaluate your answer for the following two sets of parameters.

$$A = 3, 8 \text{ V}$$
$$\omega = 30 \times 10^3, 60 \times 10^3 \text{ rad/s}$$
$$R_1 = 100, 1000\ \Omega$$
$$R_2 = 1000, 100\ \Omega$$
$$L = 10, 100 \text{ mH}.$$

The first set should yield $v_o = 2.61e^{j0.294}$.