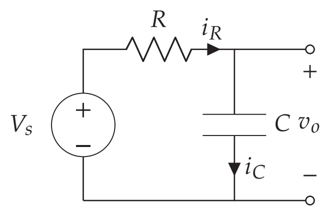
3.5 Problems

SJ

Problem 3.1 NP For the RC circuit diagram below, perform a circuit analysis to solve for the steady state voltage $v_o(t)$ if $V_S(t) = A\ \sin \omega t$, where $A \in \mathbb{R}$ is a given amplitude and $\omega \in \mathbb{R}$ is a given angular frequency. Use a *sine* phasor in the problem. Write your answer as a single sine phasor in *polar* form. Evaluate your answer for the following two sets of parameters.

$$A = 2, 5\ \text{V}$$
$$\omega = 10 \times 10^3, 20 \times 10^3\ \text{rad/s}$$
$$R = 100, 1000\ \Omega$$
$$C = 100, 10\ \text{nF}.$$

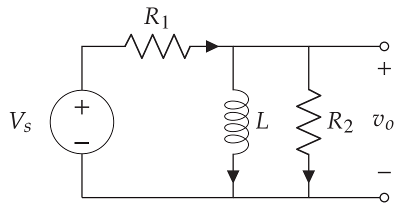
The first set should yield $v_o = 1.99e^{-j0.0997}$.



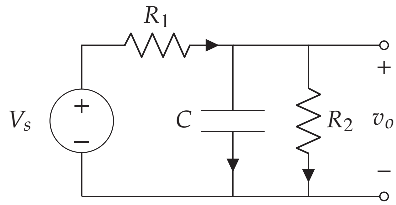
Problem 3.2 ZK For the circuit diagram below, perform a complete circuit analysis to solve for the steady state voltage $v_o(t)$ if $V_s(t) = A\ \sin \omega t$, where $A \in \mathbb{R}$ is a given amplitude and $\omega \in \mathbb{R}$ is a given angular frequency. Use a *sine* phasor in the problem. Write your answer as a single sine phasor in *polar* form. Evaluate your answer for the following two sets of parameters.

$$A = 3, 8\ \text{V}$$
$$\omega = 30 \times 10^3, 60 \times 10^3\ \text{rad/s}$$
$$R_1 = 100, 1000\ \Omega$$
$$R_2 = 1000, 100\ \Omega$$
$$L = 10, 100\ \text{mH}.$$

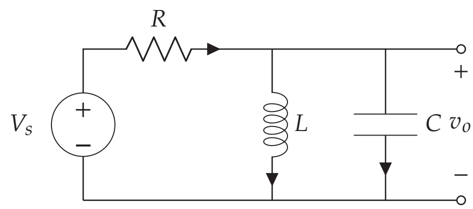
The first set should yield $v_o = 2.61e^{j0.294}$.



Problem 3.3 IW For the circuit diagram below, solve for the steady state voltage $v_o(t)$ if $V_s(t) = Ae^{j\phi}$, where $A \in \mathbb{R}$ is a given input amplitude and $\phi \in \mathbb{R}$ is a given input phase. Write your answer as a single phasor in *polar* form (you may use intermediate variables in this final form as long as they're clearly stated).



Problem 3.4 TQ For the circuit diagram below, solve for the steady state output voltage $v_o(t)$ if $V_S(t) = A\cos(\omega t)$. *Do* write V_S and the impedance of each element in phasor/polar form. *Do not* substitute V_S or the impedance of each element into your expression for $v_o(t)$. Recommendation: use a divider rule.



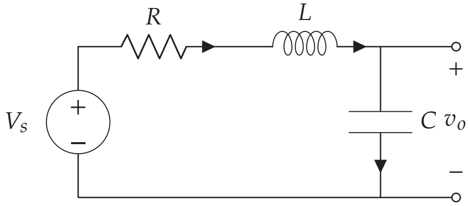
Problem 3.5 HZ For the circuit diagram below, solve for the steady state output voltage $v_o(t)$ if $V_s(t) = 3\ \sin(10t)$. Use a *sine* phasor in the problem. Write your answer as a single sine phasor in *polar* form. Evaluate your answer for the following two sets of parameters.

$$R = 10, 10^6\ \Omega$$

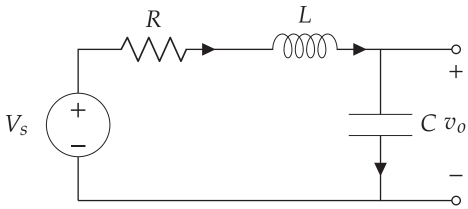
$$L = 500, 50\ \text{mH}$$

$$C = 100, 10\ \mu\text{F}.$$

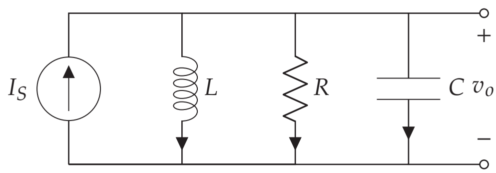
The first set should yield $v_o = 3.01e^{-j0.0100}$.



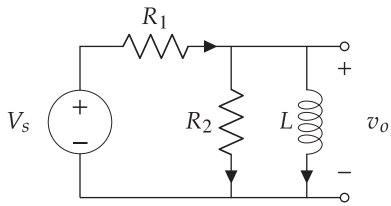
Problem 3.6 M8 For the circuit diagram below, solve for $v_o(t)$ if $V_s(t) = A \sin \omega t$, where $A = 2$ V is the given amplitude and $\omega \in \mathbb{R}$ is a given angular frequency. Let $R = 50\ \Omega$, $L = 50$ mH, and $C = 200$ nF. Find the steady-state ratio of the output amplitude to the input amplitude A for $\omega = \{5000, 10000, 50000\}$ rad/s. Plot the steady-state ratio as a function of ω in MATLAB, Python, or Mathematica. This circuit is called a **low-pass filter—**explain why this makes sense. Note that using impedance methods for steady state analysis makes this problem much easier than the transient analysis of this circuit in Exercise 2.6.



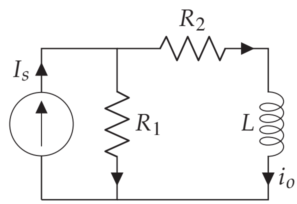
Problem 3.7 X8 For the circuit diagram below, perform a circuit analysis to solve for the steady state voltage $v_o(t)$ if $I_S = Ae^{j0}$, where $A > 0$ is a given amplitude. Identify all impedance values in the circuit, but express your answer in terms of impedances (i.e. don't substitute for them in your final expression).



Problem 3.8 A4 For the circuit diagram below, perform a complete circuit analysis to solve for the steady state voltage $v_o(t)$ if $V_s(t) = Ae^{j\phi}$, where $A \in \mathbb{R}$ is a given input amplitude and $\phi \in \mathbb{R}$ is a given input phase. Write your answer as a single phasor in *polar* form. *Hint:* consider using a divider rule, but be wary of the parallel impedances for R_2 and L!



Problem 3.9 E0 For the circuit diagram below, perform a complete circuit analysis to solve for the steady state current $i_o(t)$ if $I_S(t) = Ae^{j\phi}$, where $A \in \mathbb{R}$ is a given input amplitude and $\phi \in \mathbb{R}$ is a given input phase. Write your answer as a single phasor in *polar* form. *Hint:* consider using the current divider rule, but be wary of the series impedances for R_2 and L! Clearly define any new constants you introduce.



4 Nonlinear and Multiport Circuit Elements

5P

Thus far, we have considered only *one-port*, *linear* circuit elements. One-port elements have two terminals. Linear elements have voltage-current relationships that can be described by linear algebraic or differential equations.

Multi-port elements are those that have more than one port. In this chapter, we will consider several multi-port elements: transformers (two-port), transistors (two-port), and opamps (four-port).

Nonlinear elements have voltage-current relationships that cannot be described by a linear algebraic or differential equations. The convenient impedance methods of chapter 3 apply only to linear circuits, so we must return to the differential equation-based analysis of chapter 2. In this chapter, we will consider several nonlinear circuits containing three different classes of nonlinear elements: diodes, transistors, and opamps.

A great number of the most useful circuits today include multi-port and nonlinear elements. Tasks such as ac-dc conversion, switching, amplification, and isolation require these elements.

We explore only the fundamentals of each element considered and present basic analytic techniques, but further exploration in Horowitz and Hill (2015), Agarwal and Lang (2005), and Ulaby, Maharbiz, and Furse (2018) is encouraged.

4.1 Transformers

Electrical transformers are *two-port linear* elements that consist of two tightly coupled coils of wire. Due to the coils' magnetic field interaction, time-varying current through one side induces a current in the other (and vice-versa).

Let the terminals on the **primary (source) side** have label "1" and those on the **secondary (load) side** have label "2," as shown in figure 4.1. These devices are very efficient, so we often assume no power loss. With this assumption, the power into the transformer must sum to zero, giving us one voltage-current relationship:

$$\mathcal{P}_1 + \mathcal{P}_2 = 0$$
$$v_1 i_1 = -v_2 i_2.$$

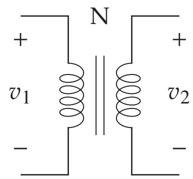


Figure 4.1. Circuit symbol for a transformer with a core. Those with "air cores" are denoted with a lack of vertical lines.

Note that with two ports, we need two elemental equations to fully describe the voltage-current relationships. Another equation can be found from the magnetic field interaction. Let N_1 and N_2 be the number of turns per coil on each side and $N \equiv N_2/N_1$. Then

$$\frac{v_1}{v_2} = \frac{1}{N}.$$

These two equations can be combined to form the following elemental equations.

Definition 4.1

$$v_2 = N v_1 \qquad i_2 = -\frac{1}{N} i_1$$

So we can **step-down** voltage if $N < 1$. This is better, in some cases, than the voltage divider because it does not dissipate much energy. However, transformers can be bulkier and somewhat nonlinear; moreover, they *only work for ac signals*. Note that when we step-down voltage, we step-up current due to our power conservation assumption.