## 4.1 Transformers

Electrical transformers are *two-port linear* elements that consist of two tightly coupled coils of wire. Due to the coils' magnetic field interaction, time-varying current through one side induces a current in the other (and vice-versa).

Let the terminals on the **primary (source) side** have label "1" and those on the **secondary (load) side** have label "2," as shown in figure 4.1. These devices are very efficient, so we often assume no power loss. With this assumption, the power into the transformer must sum to zero, giving us one voltage-current relationship:

$$\mathcal{P}_1 + \mathcal{P}_2 = 0$$

$$v_1 i_1 = -v_2 i_2.$$

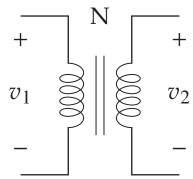



Figure 4.1. Circuit symbol for a transformer with a core. Those with "air cores" are denoted with a lack of vertical lines.

Note that with two ports, we need two elemental equations to fully describe the voltage-current relationships. Another equation can be found from the magnetic field interaction. Let $N_1$ and $N_2$ be the number of turns per coil on each side and $N \equiv N_2/N_1$. Then

$$\frac{v_1}{v_2} = \frac{1}{N}.$$

These two equations can be combined to form the following elemental equations.

Definition 4.1

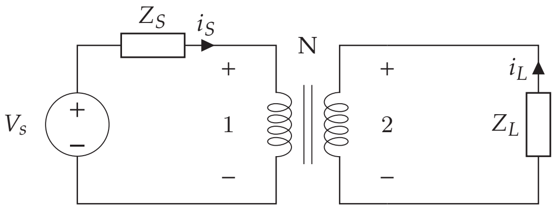
$$v_2 = N v_1 \quad i_2 = -\frac{1}{N} i_1$$

So we can **step-down** voltage if $N < 1$. This is better, in some cases, than the voltage divider because it does not dissipate much energy. However, transformers can be bulkier and somewhat nonlinear; moreover, they *only work for ac signals*. Note that when we step-down voltage, we step-up current due to our power conservation assumption.

If $N > 1$ we can **step-up** voltage. Voltage dividers cannot do this! It is not amplification, however, because power is conserved—we simultaneously step-down current. So with a transformer, we can *freely trade ac voltage and current*.

**Example 4.1**

Given the circuit shown, what is the effective impedance of $Z_L$ on the source side?




Since the effective impedance on the primary side of the transformer is, by definition,

$$Z_1 = v_1/i_1,$$

we can use the transformer elemental equations to write that in terms of the voltage and current on the secondary side:

$$Z_1 = \frac{v_2/N}{-i_2 N}$$
$$= -\frac{1}{N^2} \cdot \frac{v_2}{i_2}.$$

Let's write that in terms of the voltage and current of the load:

$$Z_1 = \frac{1}{N^2} \cdot \frac{v_L}{i_L}.$$

From the definition of the impedance of an element, $v_L/i_L = Z_L$, so

$$Z_1 = \frac{1}{N^2} Z_L.$$

This is a useful result. We say that the transformer *reflects* the impedance $Z_L$ through the factor $1/N^2$.

## 4.2 Diodes

Diodes are *single-port nonlinear* elements that, approximately, conduct current in only one direction. We will consider the ubiquitous **semiconductor diode**, varieties of which include the **light-emitting diode (LED)**, **photodiode** (for light sensing), **Schottky diode** (for fast switching), and **Zener diode** (for voltage regulation). See figure 4.2 for corresponding circuit symbols.

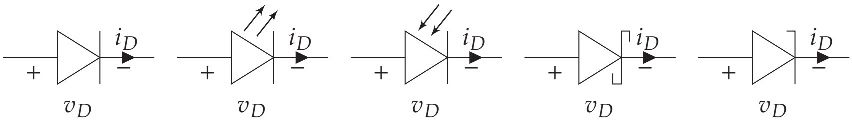


Figure 4.2. Diode symbols. From left to right, the generic symbol, LED, photodiode, Schottky, Zener.

In most cases, we use the diode to conduct current in one direction and block reverse current.[1] When conducting current in its forward direction, it is said to have **forward-bias**; when blocking current flow in its reverse direction, it is said to have **reverse-bias**. If the reverse **breakdown voltage** is reached, current will flow in the reverse direction. It is important to check that a circuit design does not subject a diode to its breakdown voltage, except in special cases (e.g. when using a Zener diode).

We begin with a nonlinear model of the voltage-current $v_D$–$i_D$ relationship. Let

- $I_s$ be the *saturation current* (typically $\sim 10^{-12}$ A) and
- $V_{\text{TH}} = k_b T/e$ be the *thermal voltage* (at room temperature $\sim 25$ mV) with[2]
  - $k_b$ the Boltzmann constant,
  - $e$ the fundamental charge, and
  - $T$ the diode temperature.

Definition 4.2

Let the nonlinear diode model be

$$i_D = I_s(\exp(v_D/V_{\text{TH}}) - 1).$$

See figure 4.3 for a plot of this function. One can analyze circuits with diodes using the methods of chapter 2 and definition 4.2 as the diode's elemental equation.

1. The paradigmatic exception is the Zener diode, which is typically used in reverse bias in order to take advantage of its highly stable reverse bias voltage over a large range of reverse current. We will not consider this application here.

2. Unless otherwise specified, it is usually reasonable to assume room-temperature operation.