4.2 Diodes

Diodes are *single-port nonlinear* elements that, approximately, conduct current in only one direction. We will consider the ubiquitous **semiconductor diode**, varieties of which include the **light-emitting diode (LED)**, **photodiode** (for light sensing), **Schottky diode** (for fast switching), and **Zener diode** (for voltage regulation). See figure 4.2 for corresponding circuit symbols.

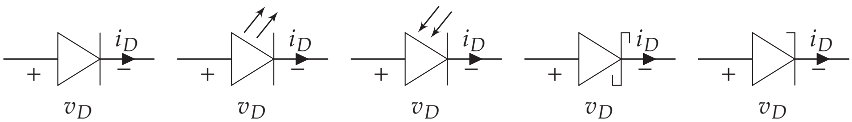


Figure 4.2. Diode symbols. From left to right, the generic symbol, LED, photodiode, Schottky, Zener.

In most cases, we use the diode to conduct current in one direction and block reverse current.[1] When conducting current in its forward direction, it is said to have **forward-bias**; when blocking current flow in its reverse direction, it is said to have **reverse-bias**. If the reverse **breakdown voltage** is reached, current will flow in the reverse direction. It is important to check that a circuit design does not subject a diode to its breakdown voltage, except in special cases (e.g. when using a Zener diode).

We begin with a nonlinear model of the voltage-current v_D–i_D relationship. Let

- I_s be the *saturation current* (typically $\sim 10^{-12}$ A) and
- $V_{\text{TH}} = k_b T/e$ be the *thermal voltage* (at room temperature ~ 25 mV) with[2]
  - k_b the Boltzmann constant,
  - e the fundamental charge, and
  - T the diode temperature.

Definition 4.2

Let the nonlinear diode model be

$$i_D = I_s(\exp(v_D/V_{\text{TH}}) - 1).$$

See figure 4.3 for a plot of this function. One can analyze circuits with diodes using the methods of chapter 2 and definition 4.2 as the diode's elemental equation.

1. The paradigmatic exception is the Zener diode, which is typically used in reverse bias in order to take advantage of its highly stable reverse bias voltage over a large range of reverse current. We will not consider this application here.

2. Unless otherwise specified, it is usually reasonable to assume room-temperature operation.

A nonlinear set of equations results, which typically require numerical solution techniques.

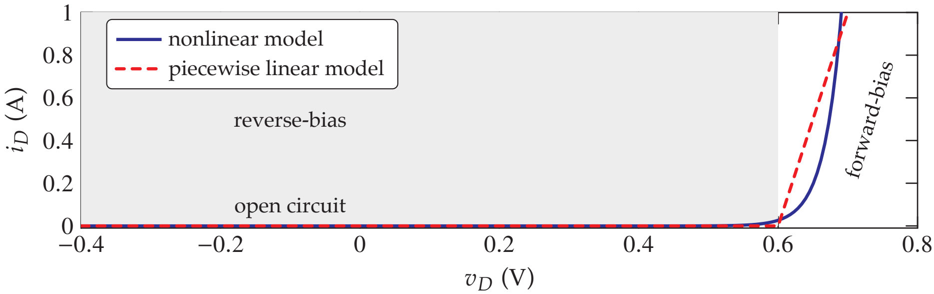


Figure 4.3. The voltage-current relationship in the nonlinear and piecewise linear models. In the figure, $R_d = 0.1\ \Omega$.

4.2.1 A Piecewise Linear Model

An **ideal diode** is one that is a perfect insulator (open circuit, $i_D = 0$) for $v_D < 0$ conductor for $v_D > 0$. We use the symbol shown in figure 4.4 for the ideal diode. At times, the ideal diode is sufficient to model a diode; often, however, we prefer a more accurate model that is piecewise linear.

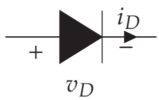


Figure 4.4. Circuit symbol for an ideal diode. Note that this is a nonstandard use of this symbol.

The **piecewise linear model** is shown in figure 4.5. It includes an ideal diode in series with a fixed voltage drop of 0.6 V and a resistor with resistance R_d. This approximates the nonlinear model with two linear approximations.

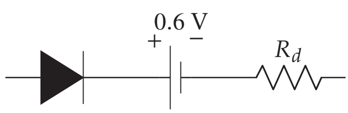


Figure 4.5. Piecewise linear diode.

Definition 4.3

Let a piecewise linear diode model be

$$i_D = \begin{cases} 0 & \text{for } v_D \leq 0.6\text{ V} \\ \dfrac{v_D - 0.6\text{ V}}{R_d} & \text{for } v_D > 0.6\text{ V} \end{cases}$$

See figure 4.3 for a plot of this function and a comparison to the nonlinear model. The slope in forward-bias is $1/R_d$. This model's effectiveness is highly dependant on R_d, so an **operating point** must be chosen and R_d chosen to match most closely with the nonlinear model near that operating point.

4.2.2 Method of Assumed States

The **method of assumed states** is a method for using linear circuit analysis to analyze circuits with nonlinear components. The method is summarized in the following steps.

1. Begin at the initial time $t = 0$.
2. Replace each diode in the circuit diagram with the piecewise linear diode model.
3. Proceed with the circuit analysis of chapter 2, ignoring the elemental equations for the ideal diodes D_i. Your system of equations will have unknown ideal diode current i_{D_i} and voltage v_{D_i}. Simplify it to the extent possible.
4. Guess the current state of each ideal diode: `ON` or `OFF`. For each ideal diode D_i guessed to be `ON`,

   $$\text{set } v_{D_i} = 0 \quad \text{and assume that} \quad i_{D_i} > 0.$$

   For each ideal diode assumed to be `OFF`,

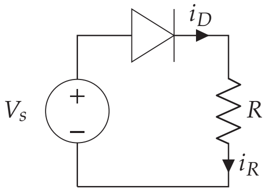
   $$\text{set } i_{D_i} = 0 \quad \text{and assume that} \quad v_{D_i} < 0.$$

   For n diodes in the circuit, there are 2^n possibilities at each moment in time. Guess just one to start.
5. If *even one diode* violates its assumption from above, dismiss the results and return to step 4 and choose a different combination of assumed states (consider flipping the assumptions on those diodes that violated the old assumptions).
6. If *not even one diode* violates its assumptions, *this is the correct solution for this moment in time*.
7. This solution is valid for as long as its assumptions are valid. Once they fail, go back to step 4.

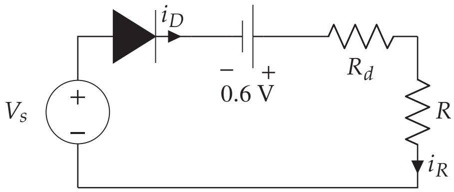
Since impedance methods are valid only for *linear* circuits, steady-state analyses should proceed with the same process outlined above. With a periodic input, a periodic (steady) solution may emerge.

Example 4.2

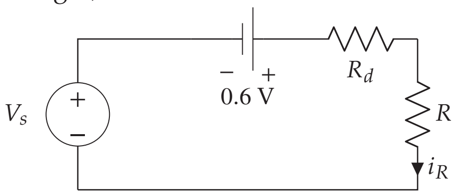
Given the circuit shown with voltage source $V_s(t) = 3\cos 2\pi t$, what is the output v_R? Explain why this might be called a "half-wave rectifier." Let $R = 10\ \Omega$.



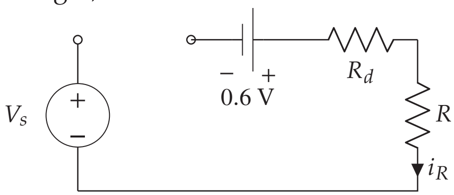
Let's use the piecewise linear model, which can be drawn as shown. We use the method of assumed states. First, subcircuits.



At right, we have the subcircuit with the *on* state of the ideal diode.



At right, we have the subcircuit with the *off* state of the ideal diode.



Now, to analyze each circuit. The on circuit, if we consider the dc voltage source to be an offset of the ac source, it is a voltage divider:

$$v_R = \frac{R}{R + R_d}(V_s - 0.6).$$

The value of R_d is tbd. Let's use a thought experiment to decide what range of current for which we might need R_d to be valid. We know that

$$\begin{aligned} i_D &= i_R \\ &= v_R/R. \end{aligned}$$

If $R_d = 0$, the maximum value of v_R is $3 - 0.6 = 2.4$ V. Therefore,

$$\begin{aligned} i_{D\text{max}} &= 2.4/10 \\ &= 0.24 \text{ A}. \end{aligned}$$

From figure 4.3, we can choose a reasonable value of $R_d = 0.2\ \Omega$. Now that we have R_d, so we now have our on solution:

$$\begin{aligned} v_R &= \frac{10}{10+0.2}(3\cos 2\pi t - 0.6) \\ &\approx 2.94\cos 2\pi t - 0.588. \end{aligned}$$

The off solution for v_R is $v_R = i_R R = (0)R = 0$ V. To determine which solution is valid, we need to determine when the on solution predicts $i_D < 0$. Since $i_D = i_R = v_R/R$,

$$i_D = 0.294\cos 2\pi t - 0.0588$$

which is positive for an interval each period. The endpoints of the interval correspond to

$$\begin{aligned} 0 &= 0.294\cos 2\pi t - 0.0588 \\ t &= \pm\frac{1}{2\pi}\arccos 0.2 \pm T \\ &= \pm 0.218\ \pm T \text{ sec} \end{aligned}$$

where $T \in \mathbb{N}_0$ is the period. The function is positive between these points in each period. Therefore, for all positive intervals, the on solution is correct, and for all negative intervals, the off solution is correct.

Consider figure 4.6. The voltage output is always positive, but the effects of the non-ideality of the diode are apparent with the voltage offset and the (slight) scaling.

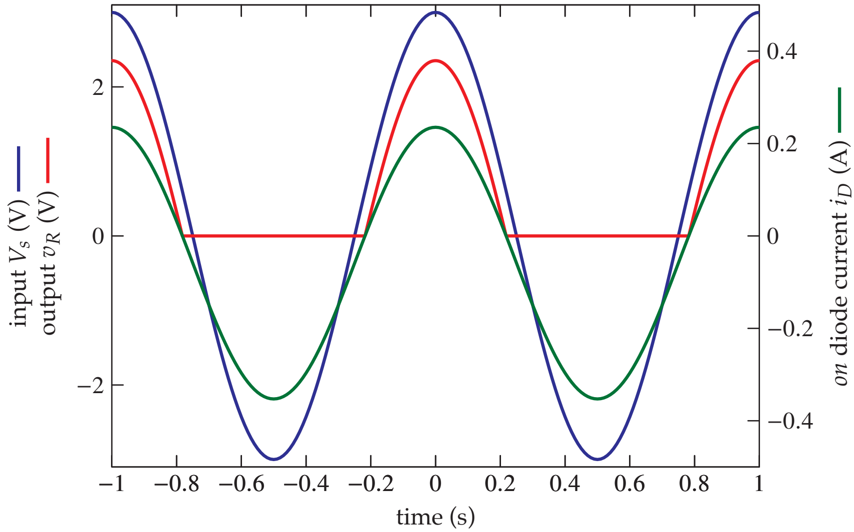


Figure 4.6. The input and output voltage of the half-wave rectifier circuit. Note that the "on" diode subcircuit is valid for $i_D > 0$ and the "off" diode circuit is valid for $i_D < 0$.

This circuit is called the half-wave rectifier because it gives approximately the upper-half of the wave at its output. This introduces a dc offset. With some filtering, this can be used as an ac-to-dc rectifier. Note that typically a *full-wave bridge rectifier* is used for this application.

4.2.3 An Algorithm for Determining R_d

The piecewise linear approximation of the exponential diode current will never be great, but we can at least try to choose R_d in a somewhat optimal way, recognizing that when highly accurate results are required, there's no substitute for the nonlinear model.

Consider the algorithm of figure 4.7. Initially set to zero the diode resistances R_{d_i} of each resistor. Solve for each diode current $i_{D_i}(t)$, then use this to find $v_{D_i}(t)$ from the *nonlinear* model of definition 4.2:

$$v_{D_i}(t) = V_{\mathrm{TH}} \ln(i_{D_i}(t)/I_s + 1).$$

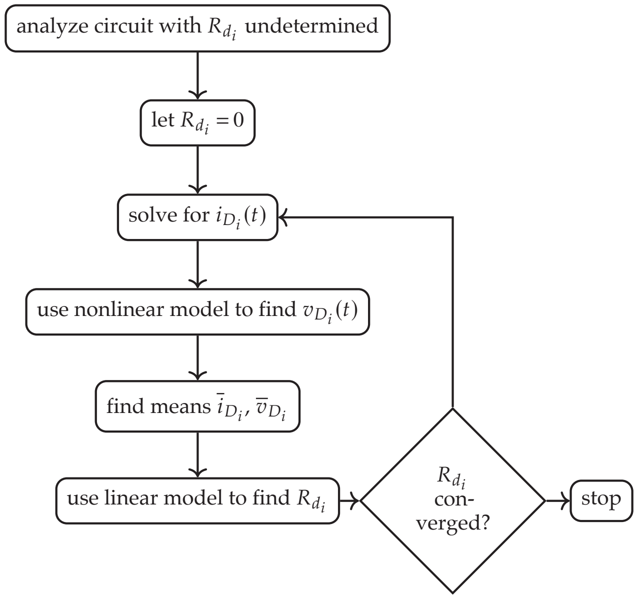


Figure 4.7. An algorithm for determining R_{d_i}.

Now take the means of these signals (assuming steady state oscillation) over a period T, excluding the time T_0 during which the diode voltage was in reverse-bias:[3]

$$\bar{i}_{D_i} = \frac{1}{T - T_0} \int_{t_0}^{t_0+T} i_{D_i}(\tau) d\tau \tag{4.1}$$

$$\bar{v}_{D_i} = \frac{1}{T - T_0} \int_{t_0}^{t_0+T} v_{D_i}(\tau) d\tau. \tag{4.2}$$

Now us the piecewise linear model of definition 4.3 to estimate R_{d_i}:

$$R_{d_i} = \frac{\bar{v}_{D_i} - 0.6V}{\bar{i}_{D_i}}.$$

We can use this estimate of R_{d_i} to re-analyze the circuit and repeat the same process of deriving a new estimate of R_{d_i}. This process should converge on an estimate of R_{d_i} that is in some sense optimal.

Note that if, during this iterative process, one finds $\bar{v}_{D_i} < 0.6$ V, a *negative* R_d will result. At this point, a couple different reasonable approaches can be taken:

3. Note that if T_0 is ignored, our estimate of R_d will include the effects of time during which no current is flowing and the diode is in reverse-bias, during which time R_d is not applicable.

1. just use $R_{d_i} = 0$ or
2. use some reasonably central value of $\overline{v}_{D_i} > 0.6$ V.

The second case is preferred if $v_{D_i}(t)$ spends much time above 0.6 V. But usually, if it spends much time, the mean $\overline{v}_{D_i}$ should be great enough to avoid this situation. Circuits that tend to express this behavior are those with high impedance and correspondingly low currents.

4.3 MOSFETs

40

A metal–oxide–semiconductor field-effect transistor (MOSFET) is a *two-port*, *nonlinear* circuit element that lies at the heart of digital electronics, with sometimes millions integrated into a single microprocessor. They are the dominant type of **transistor**, a class of elements that includes the **bipolar junction transistor (BJT)**.

MOSFETs are not just common in integrated circuits made of silicon, they are also available as discrete elements, which is the form most often encountered by the mechatronicist.

There are two primary types of MOSFET: the **n-channel** and the **p-channel**, determined by the type of semiconductor doping (negative or positive) used in the manufacturing process. These types are "opposites," so we choose to focus on n-channel, here.

Figure 4.8 displays the circuit diagram symbol for the MOSFET. There are three[4] terminals: the **gate** G, **drain** D, and **source** S. The current flowing from one terminal to another is labeled with consecutive subscripts; for instance, the current flowing from drain to source is i_{DS}. Similarly, the voltage drop across two terminals is labeled with concurrent subscripts; for instance, the voltage drop from gate to source is v_{GS}.

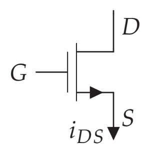


Figure 4.8. Circuit symbol for a n-channel MOSFET.

The input-output characteristics of the MOSFET are quite complex, but we may, in the first approximation, consider it to be like a *switch*. In this model, called the

4. Note that if we consider the gate-side to be the input with $i_{GS} = 0$ and v_{GS} and the drain-source-side to be the output with i_{DS} and v_{DS}, the MOSFET can be seen to be two-port.