

4.4 Operational Amplifiers



The **operational amplifier** (opamp) is the queen of analog electronic components. The opamp is a *four-port nonlinear* voltage-controlled voltage source, but it's so much more. Here are a few applications from the opamp highlight reel: summing two signals, subtracting two signals, amplifying a signal, integrating a signal, differentiating a signal, filtering a signal, isolating two subcircuits, generating periodic functions (e.g., sinusoids and square waves), and analog feedback control. Although they are nonlinear, in most applications a linear approximation is sufficiently accurate.

Figure 4.10 shows the circuit symbol for the opamp. Three terminals are displayed: **inverting input (-)** The **inverting input** is labeled with the “-” symbol. **non-inverting input (+)** The **non-inverting input** is labeled with the “+” symbol. **output** The **output** extends from the tip of the symbol, opposite the inputs.

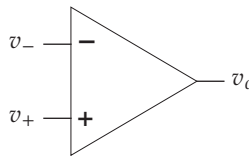


Figure 4.10. Circuit symbol for an opamp.

These comprise an input and an output port. However, there are two power supply ports that are typically suppressed in the circuit diagram. These two power supply ports are from a **differential supply**, which has a positive terminal (e.g., +12 V), symmetrically negative terminal (e.g., -12 V), and a common ground. The supply provides the opamp with external power, making it an **active** element.

When an opamp is operating in its linear mode, it outputs a voltage v_o that is A times the difference between its inputs v_+ and v_- . The **open-loop gain** A is different for every opamp, but is usually greater than 10^5 . Let's formalize this model.

Definition 4.5

An opamp's input terminals + and - draw zero current (i.e. have infinite input impedance). Let A be a positive real number. The output voltage v_o is given by

$$v_o = A(v_+ - v_-).$$

The output terminal has zero impedance.

Note that this model is equivalent to a **dependent voltage source** controlled by the input voltage difference. In fact, it is also *linearly* dependent, so linear circuit analysis techniques can be applied.⁵

The model is fairly accurate as long as $|v_o|$ is less than the maximum power source voltage. Due to the high open-loop gain, the difference in input voltage is highly restrictive for linear operation. This turns out not to be difficult to achieve, but does lead to a convenient approximation during analysis that applies most of the time:

$$v_+ \approx v_- \quad (4.3)$$

because other voltages in the circuit are typically much larger than the input voltage difference. We cannot, however, make this assumption unless (1) the opamp is operating in linear mode and (2) the opamp is part of a circuit that connects its output—via a wire or circuit elements—back to its inverting input (–).

This second condition is called **negative feedback** and is used in most opamp circuits for several reasons, the most important of which is that equation (4.3) holds due to the virtual guarantee of linear operation in this case.

4.4.1 Negative Feedback

We can think of negative feedback as continuously adjusting the output such that equation (4.3) is approximately true.⁶ Consider the feedback of v_o to the inverting input (called **unity feedback**), as shown in figure 4.11a, such that the output equation can be transformed as follows:

$$\begin{aligned} v_o &= A(v_+ - v_-) \\ &= A(v_i - v_o) \quad \Rightarrow \\ v_o &= \frac{A}{1+A}v_i \end{aligned}$$

Since $A \gg 1$, $v_o \approx v_i$. In other words, for *negative unity feedback*, v_o follows v_i . For this reason, this particular opamp circuit is called a **voltage follower**. Let's consider negative feedback's effect on the difference in input voltage:

$$\begin{aligned} v_+ - v_- &= v_+ - \frac{A}{1+A}v_+ \\ &\approx 0. \end{aligned}$$

5. Note that, while the transistor can be considered a nonlinear dependent current source, the opamp can be considered a linear dependent voltage source. However, we can easily adapt an opamp circuit to behave as a linear dependent current source, so typically the opamp is still preferred.

6. Negative feedback is considered in detail in courses on control theory. The opamp was used extensively for feedback control until low-cost, high-performance digital microcontrollers became available. Opamp-based feedback control is now called *analog feedback control*, which still has certain applications.

This is equivalent to equation (4.3). That is, for negative feedback, the input voltages are nearly equal: $v_+ \approx v_-$. This is control theory—this is how we make a system behave the way we want! In this instance, the **loop gain**—the effective gain from v_i to v_o —is one. This same principle applies when elements such as resistors and capacitors are placed in the feedback path. The resulting loop gain can be nonunity and respond dynamically to the signal.

4.4.2 Non-Inverting Opamp Circuit

The **non-inverting opamp circuit** is shown in figure 4.11b. Let’s analyze the circuit to find $v_o(v_i)$. We begin with the KVL expression for v_o in terms of v_{R_1} and v_{R_2} :

$$v_o = v_{R_1} + v_{R_2}.$$

Let’s use Ohm’s law to write:

$$v_o = i_{R_1}R_1 + i_{R_2}R_2.$$

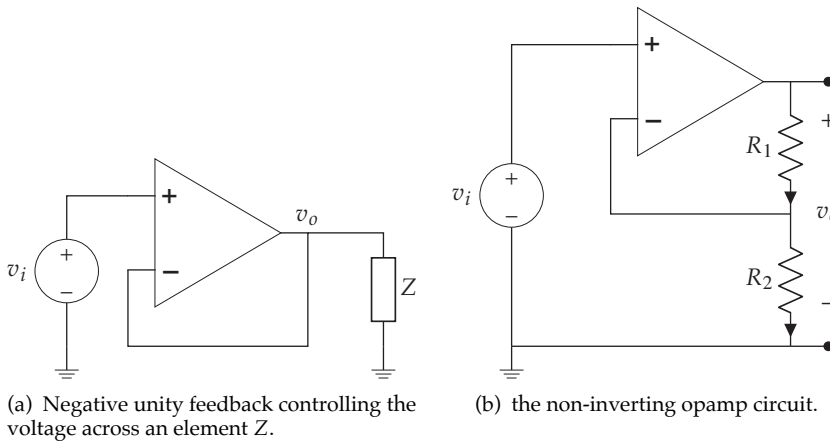


Figure 4.11. Two opamp circuits.

The KCL equation for the node between R_1 and R_2 gives

$$\begin{aligned} i_{R_1} - i_{R_2} - i_{\text{out}} &= 0 \Rightarrow \\ i_{R_1} &= i_{R_2} \Rightarrow \\ v_o &= i_{R_2}(R_1 + R_2) \Rightarrow \\ i_{R_2} &= v_o/(R_1 + R_2). \end{aligned}$$

We can write another equation for v_o from the opamp:

$$\begin{aligned} v_o &= A(v_+ - v_-) \\ &= A(v_i - v_{R_2}). \end{aligned}$$

We have an expression for i_{R_2} that can eliminate v_{R_2} with a little Ohm's law action:

$$\begin{aligned} v_o &= A(v_i - i_{R_2}R_2) \\ &= A(v_i - v_oR_2/(R_1 + R_2)) \quad \Rightarrow \\ v_o + v_oAR_2/(R_1 + R_2) &= Av_i \\ v_o &= \frac{A}{1 + AR_2/(R_1 + R_2)}v_i \\ &= \frac{(R_1 + R_2)/R_2}{(R_1 + R_2)/(R_2A) + 1}v_i. \end{aligned}$$

If $A \gg (R_1 + R_2)/R_2$, the denominator of this expression goes to 1 and we have the loop gain approximately

$$\frac{R_1 + R_2}{R_2}.$$

This gives the following input-output equation for the circuit:

$$v_o = \frac{R_1 + R_2}{R_2}v_i. \quad (4.4)$$

It is highly significant that equation (4.4) doesn't depend on A , which can be quite variable. Rather, it depends on the resistances R_1 and R_2 , only—and these are very reliable. As long as the condition


$$A \gg \frac{R_1 + R_2}{R_2}$$

is satisfied, equation (4.4) is valid.


This independence of the input-output relationship on the open-loop gain A is very common for opamp circuits. We have essentially traded gain for better linearity and gain invariance. It can be shown that this is equivalent to the assumption that $v_+ \approx v_-$. Making this assumption earlier in the analysis can simplify the process. Note that we do not use the assumption for the opamp equation $v_o = A(v_+ - v_-)$, for this would imply $v_o = 0$. Instead, in the previous analysis, we could immediately assume that $v_{R_2} = v_i$ and proceed in a similar fashion.

4.5 Problems




Problem 4.1  **RHINOCEROS** Write a one- or two-sentence response to each of the following questions and imperatives. The use of equations is acceptable when they appear in a sentence. Don't quote me (use your own words, other than technical terminology).


- Write the equivalent impedance of a resistor R and an inductor L in series. Express the result in rectangular and polar (phasor) form.
- How do you find the Norton equivalent resistance?
- Explain how a diode operates in forward-bias.
- In a MOSFET, how much current will flow from the drain D to the source S when the gate-source voltage is 0.3 V? Succinctly explain/justify.

Problem 4.2  **FLAMINGO** Write a one- or two-sentence response to each of the following questions and imperatives. The use of equations is acceptable when they appear in a sentence. Don't quote me (use your own words, other than technical terminology).

- Describe a couple differences between MOSFETs and opamps.
- If a DC source is connected to a circuit in steady state, describe how an inductor in the circuit will be operating.
- If a transformer increases an AC signal's voltage by a factor of 119, what happens to the signal's current?
- How do we determine the diode resistance for the piecewise linear model of a diode?

Problem 4.3  **ASTRINGENT** Write a one- or two-sentence response to each of the following questions and imperatives. The use of equations is acceptable when they appear in a sentence. Don't quote me (use your own words, other than technical terminology).

- If the current through an inductor is suddenly switched off, what happens?
- Let the output voltage of a resistor circuit be 5 V and the equivalent resistance 500 Ω . What is the Thevenin equivalent circuit?
- In the preceding part of this question, what is the Norton equivalent?
- When can we use impedance analysis?

Problem 4.4  **PROLONGATE** For the circuit diagram of figure 4.12, solve for $v_o(t)$ if $V_s(t) = A \cos \omega t$. Let $N = n_2/n_1$, where n_1 and n_2 are the number of turns in each coil, 1 and 2, respectively. Also let $i_L(0) = 0$ be the initial condition.