03.2 ssan.imp Impedance

With complex representations for voltage and current, we can introduce the concept of **impedance**.

Definition 03 ssan.1: impedance

Impedance Z is the complex ratio of voltage v to current i of a circuit element:

$$Z = \frac{v}{i}.$$

The real part $\mathrm{Re}(Z)$ is called the **resistance** and the imaginary part $\mathrm{Im}(Z)$ is called the **reactance**. As with complex voltage and current, we can represent the impedance as a *phasor*.
Note that Definition 03 ssan.1 is a generalization of Ohm's law. In fact, we call the following expression **generalized Ohm's law**:

$$v = iZ. \tag{1}$$

Impedance of circuit elements

The impedance of each of the three passive circuit elements we've considered thus far are listed, below. Wherever it appears, ω is the angular frequency of the element's voltage and current.

resistor For a resistor with resistance R, the impedance is all real:

capacitor For a capacitor with capacitance C, the impedance is all imaginary:

inductor For an inductor with inductance L, the impedance is all imaginary:

These are represented in the complex plane in Fig. imp.1.

Combining the impedance of multiple elements

As with resistance, the impedance of multiple elements may be combined to find an **effective impedance**.
K elements with impedances Z_j connected in *series* have equivalent impedance Z_e given by the expression

$$Z_e = \sum_{j=1}^{K} Z_j. \tag{2}$$

K elements with impedances Z_j connected in *parallel* have equivalent impedance Z_e given by the expression

$$Z_e = 1/\sum_{j=1}^{K} 1/Z_j. \tag{3}$$

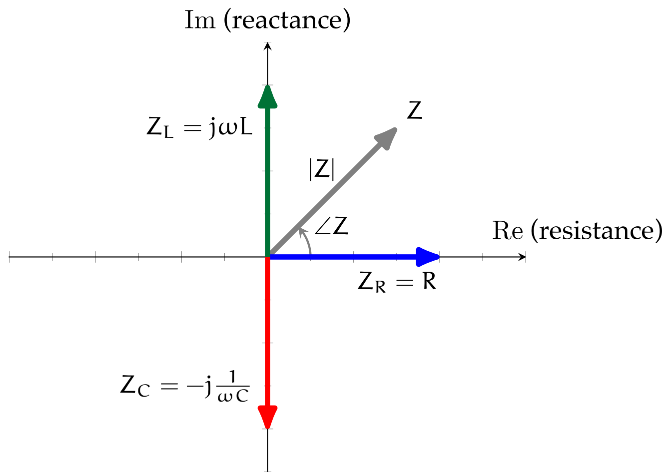

Figure imp.1: the impedance of a resistor Z_R, a capacitor Z_C, and an inductor Z_L in the complex plane.

In the special case of two elements with impedances Z_1 and Z_2,

(4)

Example 03.2 ssan.imp-1 **re: combining impedance and phasors**

Given the circuit shown with voltage source $V_s(t) = Ae^{j\phi}$, what is the total impedance at the source?.