# 04.4 nlnmul.op Operational amplifiers

The **operational amplifier** (opamp) is the queen of analog electronic components. The opamp is a *four-port nonlinear* voltage-controlled voltage source, but it's so much more. Here are a few applications from the opamp highlight reel: summing two signals, subtracting two signals, amplifying a signal, integrating a signal, differentiating a signal, filtering a signal, isolating two subcircuits, generating periodic functions (e.g. sinusoids and square waves), and analog feedback control. Although they are nonlinear, in most applications a linear approximation is sufficiently accurate.

Fig. op.1 shows the circuit symbol for the opamp. Three terminals are displayed:

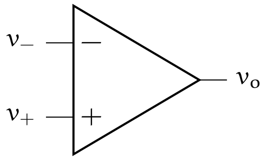



**Figure op.1:** circuit symbol for an opamp.

**inverting input** ($-$)
: The **inverting input** is labeled with the "$-$" symbol.

**non-inverting input** ($+$)
: The **non-inverting input** is labeled with the "$+$" symbol.

**output**
: The **output** extends from the tip of the symbol, opposite the inputs.

These comprise an input and an output port. However, there are two power supply ports that are typically suppressed in the circuit diagram. These two power supply ports are from a **differential supply**, which has a positive terminal (e.g. +12 V), symmetrically negative terminal (e.g. $-12$ V), and a common ground. The supply provides the opamp with external power, making it an **active** element.

When an opamp is operating in its linear mode, it outputs a voltage $v_o$ that is $A$ times the difference between its inputs $v_+$ and $v_-$. The **open-loop gain** $A$ is different for every opamp, but is usually greater than $10^5$. Let's formalize this model.

### Definition 04 nlnmul.3: opamp model

An opamp's input terminals $+$ and $-$ draw zero current (i.e. have infinite input impedance). Let $A$ be a positive real number. The output voltage $v_o$ is given by

$$v_o = A(v_+ - v_-).$$

The output terminal has zero impedance.

Note that this model is equivalent to a **dependent voltage source** controlled by the input voltage difference. In fact, it is also *linearly* dependent, so linear circuit analysis techniques can be applied.[5]

The model is fairly accurate as long as $|v_o|$ is less than the maximum power source voltage. Due to the high open-loop gain, the difference in input gain is highly restrictive for linear operation. This turns out not to be difficult to achieve, but does lead to a convenient approximation during analysis that applies most of the time:

$$v_+ \approx v_- \tag{1}$$

because other voltages in the circuit are typically much larger than the input voltage difference. We cannot, however, make this assumption unless (1) the opamp is operating in linear mode and (2) the opamp is part of a circuit that connects its output—via a wire or circuit elements—back to its inverting input ($-$). This second condition is called **negative feedback** and is used in most opamp circuits for several reasons, the most important of which is that Eq. 1 holds due to the virtual guarantee of linear operation in this case.

5. Note that, while the transistor can be considered a nonlinear dependent current source, the opamp can be considered a linear dependent voltage source. However, we can easily adapt an opamp circuit to behave as a linear dependent current source, so typically the opamp is still preferred.

## Negative feedback

We can think of negative feedback as continuously adjusting the output such that Eq. 1 is approximately true.[6] Consider the feedback of $v_o$ to the inverting input (called **unity feedback**), as shown in Fig. op.2(a), such that the output equation can be transformed as follows:

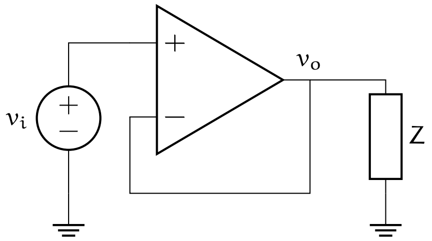
Since $A \gg 1$, $v_o \approx v_i$. In other words, *for negative unity feedback,* $v_o$ *follows* $v_i$. For this reason, this particular opamp circuit is called a **voltage follower**. Let's consider negative feedback's effect on the difference in input voltage:

This is equivalent to Eq. 1. That is, *for negative feedback, the input voltages are nearly equal:* $v_+ \approx v_-$. This is control theory—this is how we make a system behave the way we want! In this instance, the **loop gain**—the effective gain from $v_i$ to $v_o$—is one. This same principle applies when elements such as resistors and capacitors are placed in the feedback path. The resulting loop gain can be nonunity and respond dynamically to the signal.

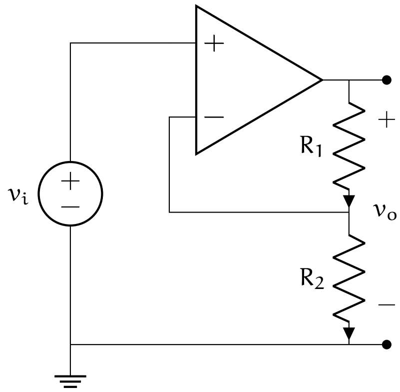
6. Negative feedback is considered in detail in courses on control theory. The opamp was used extensively for feedback control until low-cost, high-performance digital microcontrollers became available. Opamp-based feedback control is now called *analog feedback control*, which still has certain applications.

## Non-inverting opamp circuit

The **non-inverting opamp circuit** is shown in Fig. op.2(b). Let's analyze the circuit to find




(a) negative unity feedback controlling the voltage across an element $Z$.




(b) the non-inverting opamp circuit.

**Figure op.2:** two opamp circuits.

$v_o(v_i)$. We begin with the KVL expression for $v_o$ in terms of $v_{R_1}$ and $v_{R_2}$:

Let's use Ohm's law to write:

The KCL equation for the node between $R_1$ and $R_2$ gives

We can write another equation for $v_o$ from the opamp:

We have an expression for $i_{R_2}$ that can eliminate $v_{R_2}$ with a little Ohm's law action:

If $A \gg (R_1 + R_2)/R_2$, the denominator of this expression goes to 1 and we have the loop gain approximately

This gives the following input-output equation for the circuit.

**Equation 2 non-inverting opamp circuit i/o equation**

It is highly significant that Eq. 2 doesn't depend on $A$, which can be quite variable. Rather, it depends on the resistances $R_1$ and $R_2$, only—and these are very reliable. As long as the condition

$$A \gg \frac{R_1 + R_2}{R_2} \tag{3}$$

is satisfied, Eq. 2 is valid.

This independence of the input-output relationship on the open-loop gain $A$ is very common for opamp circuits. We have essentially traded gain for better linearity and gain invariance. It can be shown that this is equivalent to the assumption that $v_+ \approx v_-$. Making this assumption earlier in the analysis can simplify the process. Note that we do not use the assumption for the opamp equation $v_o = A(v_+ - v_-)$, for this would imply $v_o = 0$. Instead, in the previous analysis, we could immediately assume that $v_{R_2} = v_i$ and proceed in a similar fashion.