Resource R13 Discrete-time controllers

For reference, Table 06.1 contains Tustin equivalents for some common continuous-time controllers.

Table 06.1: Tustin equivalents for common continuous-time controllers. Usage of z is contextual, meaning a *zero* in continuous transfer functions and meaning the z-transform z in discrete transfer functions.

	phase lag/lead	PI	PID
continuous	$k\dfrac{s+z}{s+p}$	$K_p + \dfrac{K_i}{s}$	$K_p + \dfrac{K_i}{s} + K_d s$
discrete	$k\dfrac{b_0 + b_1 z^{-1}}{a_0 + a_1 z^{-1}}$	$\dfrac{b_0 + b_1 z^{-1}}{a_0 + a_1 z^{-1}}$	$\dfrac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{a_0 + a_1 z^{-1} + a_2 z^{-2}}$
differential equation	$\dfrac{dy}{dt} + py = k\left(\dfrac{dx}{dt} + zx\right)$	$y = K_p x + K_i \int_0^t x\,dt$	$y = K_p x + K_i \int_0^t x\,dt + K_d \dfrac{dx}{dt}$
difference equation	$\begin{aligned} y(n) &= -\frac{a_1}{a_0} y(n-1) \\ &+ \frac{b_0}{a_0} x(n) \\ &+ \frac{b_1}{a_0} x(n-1) \end{aligned}$	$\begin{aligned} y(n) &= -\frac{a_1}{a_0} y(n-1) \\ &+ \frac{b_0}{a_0} x(n) \\ &+ \frac{b_1}{a_0} x(n-1) \end{aligned}$	$\begin{aligned} y(n) &= -\frac{a_1}{a_0} y(n-1) \\ &- \frac{a_2}{a_0} y(n-2) \\ &+ \frac{b_0}{a_0} x(n) \\ &+ \frac{b_1}{a_0} x(n-1) \\ &+ \frac{b_2}{a_0} x(n-2) \end{aligned}$
a_0	1	1	1
a_1	$(pT-2)/(pT+2)$	-1	0
a_2			-1
b_0	$k(zT+2)/(pT+2)$	$K_p + K_i T/2$	$K_p + K_i T/2 + 2K_d/T$
b_1	$k(zT-2)/(pT+2)$	$-K_p + K_i T/2$	$K_i T - 4K_d/T$
b_2			$-K_p + K_i T/2 + 2K_d/T$