4.8 Problems

KB

Problem 4.1 5Z Let $s \in \mathbb{C}$. Use SymPy to perform a partial fraction expansion on the following expression:

$$\frac{(s+2)(s+10)}{s^4 + 8s^3 + 117s^2 + 610s + 500}.$$

Problem 4.2 IE Let $x, a_1, a_2, a_3, a_4 \in \mathbb{R}$. Use SymPy to combine the cosine and sine terms that share arguments into single sinusoids with phase shifts in the following expression:

$$a_1 \sin(x) + a_2 \cos(x) + a_3 \sin(2x) + a_4 \cos(2x)$$

Problem 4.3 KR Consider the following equation, where $x \in \mathbb{C}$ and $a, b, c \in \mathbb{R}_+$,

$$ax^2 + bx + \frac{c}{x} + b^2 = 0.$$

Use SymPy to solve for x.

Problem 4.4 G9 Let $w, x, y, z \in \mathbb{R}$. Consider the following system of equations:

$$\begin{aligned} 8w - 6x + 5y + 4z &= -20 \\ 2y - 2z &= 10 \\ 2w - x + 4y + z &= 0 \\ w + 4x - 2y + 8z &= 4. \end{aligned}$$

Use SymPy to solve the system for w, x, y, and z.

Problem 4.5 IV Consider the truss shown in figure 4.8. Use a static analysis and the method of joints to develop a solution for the force in each member F_{AC}, F_{AD}, etc., and the reaction forces using the sign convention that tension is positive and compression is negative. The forces should be expressed in terms of the applied force f_D and the dimensions w and h only. Write a program that *solves for the forces symbolically* and answers the following questions:

a. Which members are in tension?
b. Which members are in compression?
c. Are there any members with 0 nominal force? If so, which?
d. Which member (or members) has (or have) the maximum compression?
e. Which member (or members) has (or have) the maximum tension?

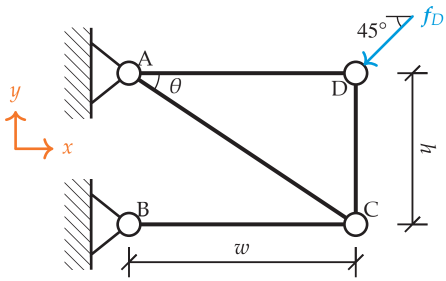

Figure 4.8. A truss with pinned joints, supported by two hinges, with an applied load f_D.

Problem 4.6 49 You are designing the truss structure shown in figure 4.9, which is to support the hanging of an external load $\boldsymbol{f}_C = -f_C\hat{\boldsymbol{j}}$, where $f_C > 0$. Your organization plans to offer customers the following options:

- Any width (i.e., $2w$)
- A selection of maximum load magnitudes $L = f_C/\alpha \in \Gamma$, where $\Gamma = \{1\text{ kN}, 2\text{ kN}, 4\text{ kN}, 8\text{ kN}, 16\text{ kN}\}$, and where α is the factor of safety

As the designer, you are to develop a design curve for the dimension h versus half-width w for each maximum load $L \in \Gamma$, under the following design constraints:

- Minimize the dimension h
- The tension in all members is no more than a given T
- The compression in all members is no more than a given C
- The magnitude of the support force at pin A is no more than a given P_A
- The magnitude of the support force at pin D is no more than a given P_D

Use a static analysis and the method of joints to develop a solution for the force in each member F_{AB}, F_{AC}, etc., and the reaction forces using the sign convention that tension is positive and compression is negative. Create a Python function that returns h as a function of w for a given set of design parameters $\{T, C, P_A, P_D, \alpha, L\}$. Use the function to create a design curve h versus $2w$ for each $L \in \Gamma$, maximum tension $T = 81$ kN, maximum compression $C = 81$ kN, maximum support A load $P_A = 50$ kN, maximum support D load $P_D = 50$ kN, and a factor of safety of $\alpha = 5$.

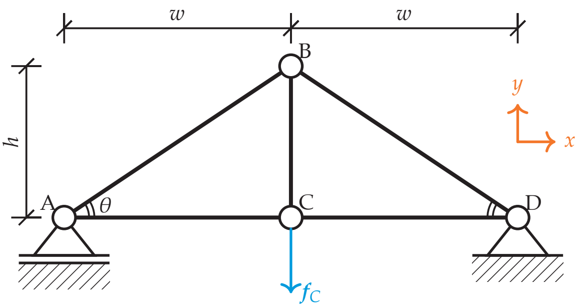

Figure 4.9. A truss with pinned joints, supported by a hinge and a floating support, with an applied load f_C.

Problem 4.7 W5 Consider an LTI system modeled by the state equation of the state-space model, equation (4.24a). A **steady state** of a system is defined as the state vector $\boldsymbol{x}(t)$ after the effects of initial conditions have become relatively small. For a constant input $\boldsymbol{u}(t) = \overline{\boldsymbol{u}}$, the constant state $\overline{\boldsymbol{x}}$ toward which the system's response decays can be found by setting the time derivative vector $\boldsymbol{x}'(t) = \boldsymbol{0}$.

Write a Python function `steady_state()` that accepts the following arguments:

- `A`: A symbolic matrix representing A
- `B`: A symbolic matrix representing B
- `u_const`: A symbolic vector representing $\overline{\boldsymbol{u}}$

The function should return `x_const`, a symbolic vector representing $\overline{\boldsymbol{x}}$.

The steady-state output converges to $\overline{\boldsymbol{y}}$ the corresponding output equation of the state-space model, equation (4.24b). Write a second Python function `steady_output()` that accepts the following arguments:

- `C`: A symbolic matrix representing C
- `D`: A symbolic matrix representing D
- `u_const`: A symbolic vector representing $\overline{\boldsymbol{u}}$
- `x_const`: A symbolic vector representing $\overline{\boldsymbol{x}}$

This function should return `y_const`, a symbolic vector representing $\overline{\boldsymbol{y}}$.

Apply `steady_state()` and `steady_output()` to the state-space model of the circuit shown in figure 4.10, which includes a resistor with resistance R, an inductor with inductance L, and capacitor with capacitance C. The LTI system is represented by equation (4.24) with state, input, and output vectors

$$\boldsymbol{x}(t) = \begin{bmatrix} v_C(t) \\ i_L(t) \end{bmatrix}, \quad \boldsymbol{u}(t) = \begin{bmatrix} V_S \end{bmatrix}, \quad \boldsymbol{y}(t) = \begin{bmatrix} v_C(t) \\ v_L(t) \end{bmatrix}$$

and the following matrices:

$$A = \begin{bmatrix} 0 & 1/C \\ -1/L & -R/L \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 1/L \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 \\ -1 & -R \end{bmatrix}, \; D = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Furthermore, let the constant input vector be

$$\overline{\boldsymbol{u}} = \begin{bmatrix} \overline{V_S} \end{bmatrix},$$

for constant $\overline{V_S}$.

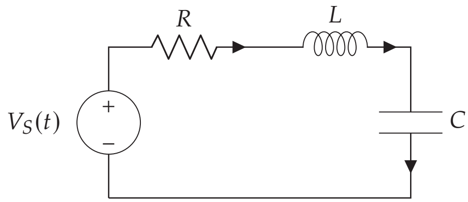

Figure 4.10. An RLC circuit with a voltage source $V_S(t)$.

Problem 4.8 8U Consider the electromechanical state-space model described in example 4.3. For a given set of parameters, input voltage, and initial conditions, the following vector-valued functions have been derived:

$$\boldsymbol{F} = \begin{bmatrix} \int_0^t v_R(t)\,dt \\ \int_0^t v_L(t)\,dt \\ \int_0^t \Omega_B(t)\,dt \\ \int_0^t \Omega_J(t)\,dt \end{bmatrix} = \begin{bmatrix} \exp(-t) \\ \exp(-t) \\ 1-\exp(-t) \\ 1-\exp(-t) \end{bmatrix}, \quad \boldsymbol{G} = \begin{bmatrix} \int_0^t i_R(t)\,dt \\ \int_0^t i_L(t)\,dt \\ \int_0^t T_B(t)\,dt \\ \int_0^t T_J(t)\,dt \end{bmatrix} = \begin{bmatrix} \exp(-t) \\ \exp(-t) \\ 1-\exp(-t) \\ \exp(-t) \end{bmatrix}$$

The instantaneous power lossed or stored by each element is given by the following vector of products:

$$\boldsymbol{\mathcal{P}}(t) = \begin{bmatrix} v_R(t) i_R(t) \\ v_L(t) i_L(t) \\ \Omega_B(t) T_B(t) \\ \Omega_J(t) T_J(t) \end{bmatrix}.$$

The energy $\mathcal{E}(t)$ of the elements, then, is

$$\boldsymbol{\mathcal{E}}(t) = \int_0^t \boldsymbol{\mathcal{P}}(t) dt.$$

Write a program that satisfies the following requirements:

a. It defines a function `power(F, G)` that returns the symbolic power vector $\boldsymbol{\mathcal{P}}(t)$ from any inputs $\boldsymbol{F}$ and $\boldsymbol{G}$

b. It defines a function `energy(F, G)` that returns the symbolic energy $\mathcal{E}(t)$ from any inputs $\boldsymbol{F}$ and $\boldsymbol{G}$ (`energy()` should call `power()`)
c. It tests the `energy()` on the specific $\boldsymbol{F}$ and $\boldsymbol{G}$ given above

Problem 4.9 FJ For the circuit and state-space model given in problem 4.7, use SymPy to solve for $\boldsymbol{x}(t)$ and $\boldsymbol{y}(t)$ given the following:

- A constant input voltage $V_S(t) = \overline{V_S}$
- Initial condition $\boldsymbol{x}(0) = \boldsymbol{0}$

Substitute the following parameters into the solution for $\boldsymbol{y}(t)$ and create numerically evaluable functions of time for each variable in $\boldsymbol{y}(t)$:

$$R = 50\ \Omega,\ L = 10 \cdot 10^{-6}\ \text{H},\ C = 1 \cdot 10^{-9}\ \text{F},\ \overline{V_S} = 10\ \text{V}.$$

Plot the outputs in $\boldsymbol{y}(t)$ as functions of time, making sure to choose a range of time over which the response is best presented. *Hint:* An appropriate amount of time is on the scale of microseconds.

5 Numerical Analysis I: Techniques

DX

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5.1 Problems

MZ

Problem 5.1 M2 asdf