1.3 Problems

K9

2 Mathematical Reasoning, Logic, and Set Theory

In order to communicate mathematical ideas effectively, **formal languages** have been developed within which **logic**, i.e. deductive (mathematical) **reasoning**, can proceed. **Propositions** are statements that can be either true $\top$ or false $\bot$. Axiomatic systems begin with statements (axioms) assumed true. **Theorems** are **proven** by deduction. In many forms of logic, like **propositional calculus** (Wikipedia 2019h), compound propositions are constructed via **logical connectives** like “and” and “or” of atomic propositions (see section 2.2). In others, like **first-order logic** (Wikipedia 2019d), there are also logical **quantifiers** like “for every” and “there exists.”

The mathematical objects and operations about which most propositions are made are expressed in terms of **set theory**, which was introduced in section 1.2 and will be expanded upon in section 2.1. We can say that mathematical reasoning is comprised of mathematical objects and operations expressed in set theory and logic allows us to reason therewith.

2.1 Introduction to Set Theory

Set theory is the language of the modern foundation of mathematics, as discussed in chapter 1. It is unsurprising, then, that it arises throughout the study of mathematics. We will use set theory extensively in chapter 3 on probability theory.

The axioms of ZFC set theory were introduced in chapter 1. Instead of proceeding in the pure mathematics way of introducing and proving theorems, we will opt for a more applied approach in which we begin with some simple definitions and include basic operations. A more thorough and still readable treatment is given by (Ciesielski 1997) and a very gentle version by (Enderton 1977).

A **set** is a collection of objects. Set theory gives us a way to describe these collections. Often, the objects in a set are numbers or sets of numbers. However, a set could represent collections of zebras and trees and hairballs. For instance, here are