2.3 Problems

H4

Problem 2.1 HARDHAT For the following, write the set described in set-builder notation.

a. $A = \{2, 3, 5, 9, 17, 33, \cdots\}$.
b. B is the set of integers divisible by 11.
c. $C = \{1/3, 1/4, 1/5, \cdots\}$.
d. D is the set of reals between -3 and 42.

Problem 2.2 ANATOMY Let $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n$. Prove the *Cauchy-Schwarz Inequality*

$$|\boldsymbol{x} \cdot \boldsymbol{y}| \leq \|\boldsymbol{x}\|\|\boldsymbol{y}\|. \tag{2.1}$$

Hint: you may find the geometric definition of the dot product helpful.

Problem 2.3 ACOUSTIC Let $\boldsymbol{x} \in \mathbb{R}^n$. Prove that

$$\boldsymbol{x} \cdot \boldsymbol{x} = \|\boldsymbol{x}\|^2. \tag{2.2}$$

Hint: you may find the geometric definition of the dot product helpful.

Problem 2.4 SUSANNA Let $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n$. Prove the *Triangle Inequality*

$$\|\boldsymbol{x} + \boldsymbol{y}\| \leq \|\boldsymbol{x}\| + \|\boldsymbol{y}\|. \tag{2.3}$$

Hint: you may find the Cauchy-Schwarz Inequality helpful.

3 Probability and Random Processes

This chapter introduces probability and random variables. Important in itself, it will also provide the basis for statistics, described in chapter 4.

3.1 Probability and Measurement

Probability theory is a well-defined branch of mathematics. Andrey Kolmogorov described a set of axioms in 1933 that are still in use today as the foundation of probability theory.[1]

We will implicitly use these axioms in our analysis. The **interpretation** of probability is a contentious matter. Some believe probability quantifies the frequency of the occurrence of some **event** that is repeated in a large number of trials. Others believe it quantifies the state of our knowledge or belief that some event will occur.

In experiments, our measurements are tightly coupled to probability. This is apparent in the questions we ask. Here are some examples.

1. How common is a given event?
2. What is the probability we will reject a good theory based on experimental results?
3. How repeatable are the results?
4. How confident are we in the results?
5. What is the character of the fluctuations and drift in the data?
6. How much data do we need?

1. For a good introduction to probability theory, see (Ash 2008) or (Jaynes et al. 2003).