Example 9.1

Consider the dynamical equation

$$x' = x^2 + r$$

with r a real constant. Sketch x' vs x for negative, zero, and positive r. Identify and classify each of the equilibria.

TODO

9.3 Simulating Nonlinear Systems

Example 9.2

Simulate a nonlinear unicycle in Python.

First, load some Python packages.

```
import numpy as np
from scipy.integrate import solve_ivp
import matplotlib.pyplot as plt
```

The state equation can be encoded via the following function `f`.

```
def f(t, x, u, c):
  dxdt = [
    x[3]*np.cos(x[2]),
    x[3]*np.sin(x[2]),
    x[4],
    1/c[0] * u(t)[0],
    1/c[1] * u(t)[1]
  ]
  return dxdt
```

The input function `u` must also be defined.

```
def u(t):
  return [
    15*(1+np.cos(t)),
    25*np.sin(3*t)
  ]
```

Define time spans, initial values, and constants

```
tspan = np.linspace(0, 50, 300)
xinit = [0,0,0,0,0]
mass = 10
inertia = 10
c = [mass,inertia]
```

Solve differential equation:

```
sol = solve_ivp(
  lambda t, x: f(t, x, u, c),
  [tspan[0], tspan[-1]],
  xinit,
  t_eval=tspan
)
```

Let's first plot the trajectory and instantaneous velocity.

```
xp = sol.y[3]*np.cos(sol.y[2])
yp = sol.y[3]*np.sin(sol.y[2])
fig, ax = plt.subplots()
plt.plot(sol.y[0],sol.y[1])
plt.quiver(sol.y[0],sol.y[1],xp,yp)
plt.xlabel('$x$')
plt.ylabel('$y$')
plt.show()
```

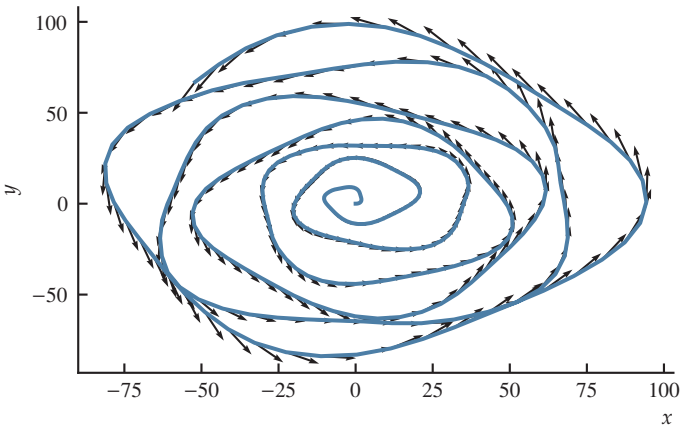

Figure 9.2. Trajectory and instantaneous velocity.

9.4 Problems