

# B Fourier and Laplace Tables



This appendix contains tables of Fourier and Laplace transforms.

## B.1 Laplace Transforms



table B.1 is a table with functions of time  $f(t)$  on the left and corresponding Laplace transforms  $L(s)$  on the right. Where applicable,  $s = \sigma + j\omega$  is the Laplace transform variable,  $T$  is the time-domain period,  $\omega_0 2\pi/T$  is the corresponding angular frequency,  $j = \sqrt{-1}$ ,  $a \in \mathbb{R}^+$ , and  $b, t_0 \in \mathbb{R}$  are constants.

Table B.1. Laplace transform identities.

function of time $t$	function of Laplace $s$
$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(s) + a_2 F_2(s)$
$f(t - t_0)$	$F(s)e^{-t_0 s}$
$f'(t)$	$sF(s) - f(0)$
$\frac{d^n f(t)}{dt^n}$	$s^n F(s) + s^{(n-1)} f(0) + s^{(n-2)} f'(0) + \dots + f^{(n-1)}(0)$
$\int_0^t f(\tau) d\tau$	$\frac{1}{s} F(s)$
$t f(t)$	$-F'(s)$
$f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau) f_2(t - \tau) d\tau$	$F_1(s) F_2(s)$
$\delta(t)$	1
$u_s(t)$	$1/s$
$u_r(t)$	$1/s^2$
$t^{n-1}/(n-1)!$	$1/s^n$
$e^{-at}$	$\frac{1}{s+a}$

$t e^{-at}$	$\frac{1}{(s+a)^2}$
$\frac{1}{(n-1)!} t^{n-1} e^{-at}$	$\frac{1}{(s+a)^n}$
$\frac{1}{a-b} (e^{at} - e^{bt})$	$\frac{1}{(s-a)(s-b)} \quad (a \neq b)$
$\frac{1}{a-b} (ae^{at} - be^{bt})$	$\frac{s}{(s-a)(s-b)} \quad (a \neq b)$
$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2 + \omega^2}$
$e^{at} \cos \omega t$	$\frac{s-a}{(s-a)^2 + \omega^2}$

## B.2 Fourier Transforms

table B.2 is a table with functions of time  $f(t)$  on the left and corresponding Fourier transforms  $F(\omega)$  on the right. Where applicable,  $T$  is the time-domain period,  $\omega_0 2\pi/T$  is the corresponding angular frequency,  $j = \sqrt{-1}$ ,  $a \in \mathbb{R}^+$ , and  $b, t_0 \in \mathbb{R}$  are constants. Furthermore,  $f_e$  and  $f_o$  are even and odd functions of time, respectively, and it can be shown that any function  $f$  can be written as the sum  $f(t) = f_e(t) + f_o(t)$ . (Hsu 1970; appendix E)



Table B.2. Fourier transform identities.

function of time $t$	function of frequency $\omega$
$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(\omega) + a_2 F_2(\omega)$
$f(at)$	$\frac{1}{ a } F(\omega/a)$
$f(-t)$	$F(-\omega)$
$f(t - t_0)$	$F(\omega) e^{-j\omega t_0}$
$f(t) \cos \omega_0 t$	$\frac{1}{2} F(\omega - \omega_0) + \frac{1}{2} F(\omega + \omega_0)$
$f(t) \sin \omega_0 t$	$\frac{1}{j2} F(\omega - \omega_0) - \frac{1}{j2} F(\omega + \omega_0)$
$f_e(t)$	$\Re F(\omega)$
$f_o(t)$	$j \Im F(\omega)$
$F(t)$	$2\pi f(-\omega)$