te^{-at}	$\frac{1}{(s+a)^2}$
$\frac{1}{(n-1)!}t^{n-1}e^{-at}$	$\frac{1}{(s+a)^n}$
$\frac{1}{a-b}(e^{at}-e^{bt})$	$\frac{1}{(s-a)(s-b)}$ $(a \neq b)$
$\frac{1}{a-b}(ae^{at}-be^{bt})$	$\frac{s}{(s-a)(s-b)}$ $(a \neq b)$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$
$e^{at} \cos \omega t$	$\frac{s-a}{(s-a)^2+\omega^2}$

B.2 Fourier Transforms

FT

table B.2 is a table with functions of time $f(t)$ on the left and corresponding Fourier transforms $F(\omega)$ on the right. Where applicable, T is the time-domain period, $\omega_0 2\pi/T$ is the corresponding angular frequency, $j = \sqrt{-1}$, $a \in \mathbb{R}^+$, and $b, t_0 \in \mathbb{R}$ are constants. Furthermore, f_e and f_0 are even and odd functions of time, respectively, and it can be shown that any function f can be written as the sum $f(t) = f_e(t) + f_0(t)$. (Hsu 1970; appendix E)

Table B.2. Fourier transform identities.

function of time t	function of frequency ω
$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(\omega) + a_2 F_2(\omega)$
$f(at)$	$\frac{1}{\lvert a \rvert} F(\omega/a)$
$f(-t)$	$F(-\omega)$
$f(t-t_0)$	$F(\omega)e^{-j\omega t_0}$
$f(t)\cos \omega_0 t$	$\frac{1}{2}F(\omega-\omega_0) + \frac{1}{2}F(\omega+\omega_0)$
$f(t)\sin \omega_0 t$	$\frac{1}{j2}F(\omega-\omega_0) - \frac{1}{j2}F(\omega+\omega_0)$
$f_e(t)$	$\Re F(\omega)$
$f_0(t)$	$j\Im F(\omega)$
$F(t)$	$2\pi f(-\omega)$

$f'(t)$	$j\omega F(\omega)$
$\dfrac{d^n f(t)}{dt^n}$	$(j\omega)^n F(\omega)$
$\int_{-\infty}^{t} f(\tau)d\tau$	$\dfrac{1}{j\omega}F(\omega) + \pi F(0)\delta(\omega)$
$-jtf(t)$	$F'(\omega)$
$(-jt)^n f(t)$	$\dfrac{d^n F(\omega)}{d\omega^n}$
$f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau)f_2(t-\tau)d\tau$	$F_1(\omega)F_2(\omega)$
$f_1(t)f_2(t)$	$\dfrac{1}{2\pi}F_1(\omega) * F_2(\omega) = \dfrac{1}{2\pi}\int_{-\infty}^{\infty} F_1(\alpha)F_2(\omega-\alpha)d\alpha$
$e^{-at}u_s(t)$	$\dfrac{1}{j\omega + a}$
$e^{-a\lvert t\rvert}$	$\dfrac{2a}{a^2+\omega^2}$
e^{-at^2}	$\sqrt{\pi/a}\, e^{-\omega^2/(4a)}$
1 for $\lvert t\rvert < a/2$, else 0	$\dfrac{a\sin(a\omega/2)}{a\omega/2}$
$te^{-at}u_s(t)$	$\dfrac{1}{(a+j\omega)^2}$
$\dfrac{t^{n-1}}{(n-1)!}e^{-at})^n u_s(t)$	$\dfrac{1}{(a+j\omega)^n}$
$\dfrac{1}{a^2+t^2}$	$\dfrac{\pi}{a}e^{-a\lvert\omega\rvert}$
$\delta(t)$	1
$\delta(t-t_0)$	$e^{-j\omega t_0}$
$u_s(t)$	$\pi\delta(\omega) + \dfrac{1}{j\omega}$
$u_s(t-t_0)$	$\pi\delta(\omega) + \dfrac{1}{j\omega}e^{-j\omega t_0}$
1	$2\pi\delta(\omega)$
t	$2\pi j\delta'(\omega)$
t^n	$2\pi j^n \dfrac{d^n\delta(\omega)}{d\omega^n}$
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
$\cos\omega_0 t$	$\pi\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0)$
$\sin\omega_0 t$	$-j\pi\delta(\omega-\omega_0) + j\pi\delta(\omega+\omega_0)$
$u_s(t)\cos\omega_0 t$	$\dfrac{j\omega}{\omega_0^2-\omega^2} + \dfrac{\pi}{2}\delta(\omega-\omega_0) + \dfrac{\pi}{2}\delta(\omega+\omega_0)$
$u_s(t)\sin\omega_0 t$	$\dfrac{\omega_0}{\omega_0^2-\omega^2} + \dfrac{\pi}{2j}\delta(\omega-\omega_0) - \dfrac{\pi}{2j}\delta(\omega+\omega_0)$

$tu_s(t)$	$j\pi\delta'(\omega) - 1/\omega^2$
$1/t$	$\pi j - 2\pi j u_s(\omega)$
$1/t^n$	$\frac{(-j\omega)^{n-1}}{(n-1)!}(\pi j - 2\pi j u_s(\omega))$
$\operatorname{sgn} t$	$\frac{2}{j\omega}$
$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\omega_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$

C Mathematics Reference

This appendix contains a reference for algebra, trigonometry, and other mathematical topics.

C.1 Quadratic Forms

The solution to equations of the form $ax^2 + bx + c = 0$ is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \tag{C.1}$$

C.1.1 H

is is accomplished by re-writing the quadratic formula in the form of the left-hand-side (LHS) of this equality, which describes factorization

$$x^2 + 2xh + h^2 = (x + h)^2. \tag{C.2}$$

C.2 Trigonometry

C.2.1 Triangle Identities

With reference to figure C.1, the *law of sines* is

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c} \tag{C.3}$$

and the *law of cosines* is

$$c^2 = a^2 + b^2 - 2ab\ \cos\gamma \tag{C.4a}$$

$$b^2 = a^2 + c^2 - 2ac\ \cos\beta \tag{C.4b}$$

$$a^2 = c^2 + b^2 - 2cb\ \cos\alpha \tag{C.4c}$$