C Mathematics Reference

This appendix contains a reference for algebra, trigonometry, and other mathematical topics.

C.1 Quadratic Forms

The solution to equations of the form $ax^2 + bx + c = 0$ is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \tag{C.1}$$

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is is accomplished by re-writing the quadratic formula in the form of the left-hand-side (LHS) of this equality, which describes factorization

$$x^2 + 2xh + h^2 = (x + h)^2. \tag{C.2}$$

C.2 Trigonometry

C.2.1 Triangle Identities

With reference to figure C.1, the *law of sines* is

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c} \tag{C.3}$$

and the *law of cosines* is

$$c^2 = a^2 + b^2 - 2ab\ \cos\gamma \tag{C.4a}$$

$$b^2 = a^2 + c^2 - 2ac\ \cos\beta \tag{C.4b}$$

$$a^2 = c^2 + b^2 - 2cb\ \cos\alpha \tag{C.4c}$$

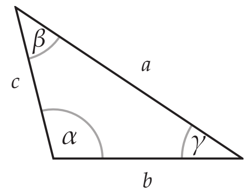

Figure C.1. Triangle for the law of sines and law of cosines.

C.2.2 Reciprocal Identities

$$\csc u = \frac{1}{\sin u} \tag{C.5a}$$

$$\sec u = \frac{1}{\cos u} \tag{C.5b}$$

$$\cot u = \frac{1}{\tan u} \tag{C.5c}$$

C.2.3 Pythagorean Identities

$$1 = \sin^2 u + \cos^2 u \tag{C.6a}$$

$$\sec^2 u = 1 + \tan^2 u \tag{C.6b}$$

$$\csc^2 u = 1 + \cot^2 u \tag{C.6c}$$

C.2.4 Cofunction Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \tag{C.7a}$$

$$\cos\left(\frac{\pi}{2} - u\right) = \sin u \tag{C.7b}$$

$$\tan\left(\frac{\pi}{2} - u\right) = \cot u \tag{C.7c}$$

$$\csc\left(\frac{\pi}{2} - u\right) = \sec u \tag{C.7d}$$

$$\sec\left(\frac{\pi}{2} - u\right) = \csc u \tag{C.7e}$$

$$\cot\left(\frac{\pi}{2} - u\right) = \tan u \tag{C.7f}$$

C.2.5 Even-Odd Identities

$$\sin(-u) = -\sin u \tag{C.8a}$$

$$\cos(-u) = \cos u \tag{C.8b}$$

$$\tan(-u) = -\tan u \tag{C.8c}$$

C.2.6 Sum-Difference Formulas (AM or Lock-In)

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v \tag{C.9a}$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v \tag{C.9b}$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v} \tag{C.9c}$$

C.2.7 Double Angle Formulas

$$\sin(2u) = 2 \sin u \cos u \tag{C.10a}$$

$$\cos(2u) = \cos^2 u - \sin^2 u \tag{C.10b}$$

$$= 2\cos^2 u - 1 \tag{C.10c}$$

$$= 1 - 2\sin^2 u \tag{C.10d}$$

$$\tan(2u) = \frac{2 \tan u}{1 - \tan^2 u} \tag{C.10e}$$

C.2.8 Power-Reducing or Half-Angle Formulas

$$\sin^2 u = \frac{1 - \cos(2u)}{2} \tag{C.11a}$$

$$\cos^2 u = \frac{1 + \cos(2u)}{2} \tag{C.11b}$$

$$\tan^2 u = \frac{1 - \cos(2u)}{1 + \cos(2u)} \tag{C.11c}$$

C.2.9 Sum-To-Product Formulas

$$\sin u + \sin v = 2\sin\frac{u+v}{2}\cos\frac{u-v}{2} \tag{C.12a}$$

$$\sin u - \sin v = 2\cos\frac{u+v}{2}\sin\frac{u-v}{2} \tag{C.12b}$$

$$\cos u + \cos v = 2\cos\frac{u+v}{2}\cos\frac{u-v}{2} \tag{C.12c}$$

$$\cos u - \cos v = -2\sin\frac{u+v}{2}\sin\frac{u-v}{2} \tag{C.12d}$$

C.2.10 Product-To-Sum Formulas

$$\sin u \sin v = \frac{1}{2}[\cos(u-v) - \cos(u+v)] \tag{C.13a}$$

$$\cos u \cos v = \frac{1}{2}[\cos(u-v) + \cos(u+v)] \tag{C.13b}$$

$$\sin u \cos v = \frac{1}{2}[\sin(u+v) + \sin(u-v)] \tag{C.13c}$$

$$\cos u \sin v = \frac{1}{2}[\sin(u+v) - \sin(u-v)] \tag{C.13d}$$

C.2.11 Two-To-One Formulas

$$A\sin u + B\cos u = C\sin(u+\phi) \tag{C.14a}$$

$$= C\cos(u+\psi) \text{ where} \tag{C.14b}$$

$$C = \sqrt{A^2+B^2} \tag{C.14c}$$

$$\phi = \arctan\frac{B}{A} \tag{C.14d}$$

$$\psi = -\arctan\frac{A}{B} \tag{C.14e}$$

C.3 Matrix Inverses

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This is a guide to inverting 1×1, 2×2, and $n \times n$ matrices.

Let A be the 1×1 matrix

$$A = \begin{bmatrix} a \end{bmatrix}.$$

The inverse is simply the reciprocal:

$$A^{-1} = \begin{bmatrix} 1/a \end{bmatrix}.$$

Let B be the 2×2 matrix

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}.$$

It can be shown that the inverse follows a simple pattern:

$$\begin{aligned} B^{-1} &= \frac{1}{\det B} \begin{bmatrix} b_{22} & -b_{12} \\ -b_{21} & b_{11} \end{bmatrix} \\ &= \frac{1}{b_{11}b_{22} - b_{12}b_{21}} \begin{bmatrix} b_{22} & -b_{12} \\ -b_{21} & b_{11} \end{bmatrix}. \end{aligned}$$

Let C be an $n \times n$ matrix. It can be shown that its inverse is

$$C^{-1} = \frac{1}{\det C} \operatorname{adj} C,$$

where adj is the **adjoint** of C.

C.4 Euler's Formulas

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Euler's formula is our bridge back-and forth between trigonomentric forms ($\cos\theta$ and $\sin\theta$) and complex exponential form ($e^{j\theta}$):

$$e^{j\theta} = \cos\theta + j\sin\theta. \tag{C.15}$$

Here are a few useful identities implied by Euler's formula.

$$e^{-j\theta} = \cos\theta - j\sin\theta \tag{C.16a}$$

$$\cos\theta = \Re(e^{j\theta}) \tag{C.16b}$$

$$= \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right) \tag{C.16c}$$

$$\sin\theta = \Im(e^{j\theta}) \tag{C.16d}$$

$$= \frac{1}{j2}\left(e^{j\theta} - e^{-j\theta}\right). \tag{C.16e}$$