C.3 Matrix Inverses

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This is a guide to inverting 1×1, 2×2, and $n\times n$ matrices.

Let A be the 1×1 matrix

$$A = \begin{bmatrix} a \end{bmatrix}.$$

The inverse is simply the reciprocal:

$$A^{-1} = \begin{bmatrix} 1/a \end{bmatrix}.$$

Let B be the 2×2 matrix

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}.$$

It can be shown that the inverse follows a simple pattern:

$$\begin{aligned} B^{-1} &= \frac{1}{\det B}\begin{bmatrix} b_{22} & -b_{12} \\ -b_{21} & b_{11} \end{bmatrix} \\ &= \frac{1}{b_{11}b_{22} - b_{12}b_{21}}\begin{bmatrix} b_{22} & -b_{12} \\ -b_{21} & b_{11} \end{bmatrix}. \end{aligned}$$

Let C be an $n\times n$ matrix. It can be shown that its inverse is

$$C^{-1} = \frac{1}{\det C}\,\text{adj}\, C,$$

where adj is the **adjoint** of C.

C.4 Euler's Formulas

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Euler's formula is our bridge back-and forth between trigonomentric forms ($\cos\theta$ and $\sin\theta$) and complex exponential form ($e^{j\theta}$):

$$e^{j\theta} = \cos\theta + j\sin\theta. \tag{C.15}$$

Here are a few useful identities implied by Euler's formula.

$$e^{-j\theta} = \cos\theta - j\sin\theta \tag{C.16a}$$

$$\cos\theta = \Re(e^{j\theta}) \tag{C.16b}$$

$$= \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right) \tag{C.16c}$$

$$\sin\theta = \Im(e^{j\theta}) \tag{C.16d}$$

$$= \frac{1}{j2}\left(e^{j\theta} - e^{-j\theta}\right). \tag{C.16e}$$