

C.3 Matrix Inverses

This is a guide to inverting 1×1 , 2×2 , and $n \times n$ matrices.

Let A be the 1×1 matrix

$$A = [a].$$

The inverse is simply the reciprocal:

$$A^{-1} = [1/a].$$

Let B be the 2×2 matrix

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}.$$

It can be shown that the inverse follows a simple pattern:

$$\begin{aligned} B^{-1} &= \frac{1}{\det B} \begin{bmatrix} b_{22} & -b_{12} \\ -b_{21} & b_{11} \end{bmatrix} \\ &= \frac{1}{b_{11}b_{22} - b_{12}b_{21}} \begin{bmatrix} b_{22} & -b_{12} \\ -b_{21} & b_{11} \end{bmatrix}. \end{aligned}$$

Let C be an $n \times n$ matrix. It can be shown that its inverse is

$$C^{-1} = \frac{1}{\det C} \text{adj } C,$$

where adj is the **adjoint** of C .

C.4 Euler's Formulas

Euler's formula is our bridge back-and-forth between trigonometric forms ($\cos \theta$ and $\sin \theta$) and complex exponential form ($e^{j\theta}$):

$$e^{j\theta} = \cos \theta + j \sin \theta. \tag{C.15}$$

Here are a few useful identities implied by Euler's formula.

$$e^{-j\theta} = \cos \theta - j \sin \theta \tag{C.16a}$$

$$\cos \theta = \Re(e^{j\theta}) \tag{C.16b}$$

$$= \frac{1}{2} (e^{j\theta} + e^{-j\theta}) \tag{C.16c}$$

$$\sin \theta = \Im(e^{j\theta}) \tag{C.16d}$$

$$= \frac{1}{j2} (e^{j\theta} - e^{-j\theta}). \tag{C.16e}$$



C.5 Laplace Transforms

The definition of the one-side Laplace and inverse Laplace transforms follow.



Definition C.1: Laplace transforms (one-sided)

Laplace transform \mathcal{L} :

$$\mathcal{L}(y(t)) = Y(s) = \int_0^{\infty} y(t)e^{-st} dt. \quad (\text{C.17})$$

Inverse Laplace transform \mathcal{L}^{-1} :

$$\mathcal{L}^{-1}(Y(s)) = y(t) = \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} Y(s)e^{st} ds. \quad (\text{C.18})$$

See table B.1 for a list of properties and common transforms.