sets.logic Logical connectives and quantifiers

In order to make compound propositions, we need to define logical connectives. In order to specify quantities of variables, we need to define logical quantifiers. The following is a form of **first-order logic** (Wikipedia, 2019d).

Logical connectives

A proposition can be either true $\top$ and false $\bot$. When it does not contain a logical connective, it is called an **atomistic proposition**. To combine propositions into a **compound proposition**, we require **logical connectives**. They are **not** ($\neg$), **and** ($\wedge$), and **or** ($\vee$). Table logic.1 is a **truth table** for a number of connectives.

Quantifiers

Logical quantifiers allow us to indicate the quantity of a variable. The **universal quantifier symbol** $\forall$ means "for all". For instance, let A be a set; then $\forall a \in A$ means "for all elements in A" and gives this quantity variable a. The **existential quantifier** $\exists$ means "there exists at least one" or "for some". For instance, let A be a set; then $\exists a \in A \ldots$ means "there exists at least one element a in A ...."

Table logic.1: a truth table for logical connectives. The first two columns are the truth values of propositions p and q; the rest are *outputs*.

		not	and	or	nand	nor	xor	xnor
p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \uparrow q$	$p \downarrow q$	$p \veebar q$	$p \Leftrightarrow q$
$\bot$	$\bot$	$\top$	$\bot$	$\bot$	$\top$	$\top$	$\bot$	$\top$
$\bot$	$\top$	$\top$	$\bot$	$\top$	$\top$	$\bot$	$\top$	$\bot$
$\top$	$\bot$	$\bot$	$\bot$	$\top$	$\top$	$\bot$	$\top$	$\bot$
$\top$	$\top$	$\bot$	$\top$	$\top$	$\bot$	$\bot$	$\bot$	$\top$