prob.prob Basic probability theory

The mathematical model for a class of measurements is called the **probability space** and is composed of a mathematical triple of a sample space Ω, σ-algebra $\mathcal{F}$, and probability measure P, typically denoted $(\Omega, \mathcal{F}, P)$, each of which we will consider in turn (Wikipedia, 2019h).

The **sample space** Ω of an experiment is the set representing all possible **outcomes** of the experiment. If a coin is flipped, the sample space is $\Omega = \{H, T\}$, where H is *heads* and T is *tails*. If a coin is flipped twice, the sample space could be

However, *the same experiment can have different sample spaces*. For instance, for two coin flips, we could also choose

We base our choice of Ω on the problem at hand.

An **event** is a subset of the sample space. That is, an event corresponds to a yes-or-no question about the experiment. For instance, event A (remember: $A \subseteq \Omega$) in the coin flipping experiment (two flips) might be $A = \{HT, TH\}$. A is an event that corresponds to the question, "Is the second flip different than the first?" A is the event for which the answer is "yes."

Algebra of events

Because events are sets, we can perform the usual set operations with them.

Example prob.prob-1 re: set operations with events

Consider a toss of a single die. We choose the sample space to be $\Omega = \{1, 2, 3, 4, 5, 6\}$. Let the following define events.

$$A \equiv \{\text{the result is even}\} = \{2, 4, 6\}$$
$$B \equiv \{\text{the result is greater than 2}\} = \{3, 4, 5, 6\}.$$

Find the following event combinations:

$$A \cup B \quad A \cap B \quad A \setminus B \quad B \setminus A \quad \overline{A} \setminus B.$$

The σ-**algebra** $\mathcal{F}$ is the collection of events of interest. Often, $\mathcal{F}$ is the set of all possible events given a sample space Ω, which is just the power set of Ω (Wikipedia, 2019h). When referring to an event, we often state that it is an element of $\mathcal{F}$. For instance, we might say an event $A \in \mathcal{F}$. We're finally ready to assign probabilities to events. We define the **probability measure** $P : \mathcal{F} \to [0, 1]$ to be a function satisfying the following conditions.

1. For every event $A \in \mathcal{F}$, the probability measure of A is greater than or equal to zero—i.e. $P(A) \geqslant 0$.
2. If an event is the entire sample space, its probability measure is unity—i.e. if $A = \Omega$, $P(A) = 1$.
3. If events $A_1, A_2, \cdots$ are disjoint sets (no elements in common), then $P(A_1 \cup A_2 \cup \cdots) = P(A_1) + P(A_2) + \cdots$.

We conclude the basics by observing four facts that can be proven from the definitions above.

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