

## prob.condition Independence and conditional probability

Two events A and B are **independent** if and only if

$$P(A \cap B) = P(A)P(B).$$

If an experimenter must make a judgment without data about the independence of events, they base it on their knowledge of the events, as discussed in the following example.

### Example prob.condition-1

### re: independence

Answer the following questions and imperatives.

1. Consider a single fair die rolled twice. What is the probability that both rolls are 6?
2. What changes if the die is biased by a weight such that  $P(\{6\}) = 1/7$ ?
3. What changes if the die is biased by a magnet, rolled on a magnetic dice-rolling tray such that  $P(\{6\}) = 1/7$ ?
4. What changes if there are two dice, biased by weights such that for each  $P(\{6\}) = 1/7$ , rolled once, both resulting in 6?
5. What changes if there are two dice, biased by magnets, rolled together?



### Conditional probability

If events  $A$  and  $B$  are somehow **dependent**, we need a way to compute the probability of  $B$  occurring given that  $A$  occurs. This is called the **conditional probability** of  $B$  given  $A$ , and is denoted  $P(B | A)$ . For  $P(A) > 0$ , it is defined as

$$P(B | A) = \frac{P(A \cap B)}{P(A)}. \tag{1}$$

We can interpret this as a restriction of the sample space  $\Omega$  to  $A$ ; i.e. the new sample space  $\Omega' = A \subseteq \Omega$ . Note that if  $A$  and  $B$  are independent, we obtain the obvious result:



#### Example prob.condition-2

re: dependence

Consider two unbiased dice rolled once. Let events  $A = \{\text{sum of faces} = 8\}$  and  $B = \{\text{faces are equal}\}$ . What is the probability the faces are equal given that their sum is 8?



